Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol.48, 2015
www.iiste.org

Renormalization quantum Fields and the Big Bang

Malik Matwi
Damascus University, Syria
malik.matwi@hotmail.com
2014-2015

Abstract

In the classical Physics, the particles are described by their masses, charges, spins, number of particles, …., which are constants values. But that was not exactly in the quantum Physics, in which, the particles properties like masses, charges, spins, number of particles, …, are not constants values, they change in the interactions.

So we need unchanged properties in the interactions, we need that to know how we describe the nature according to the quantum principles. For that the quantum fields theory was born, It is a marriage between the quantum mechanics and the Symmetries, like the space XYZ points Symmetry. In general the Symmetries are described by continuous transformations {U} form groups, to satisfy the Symmetries, the interactions must preserve them. Therefore the Lagrange structure $L(\partial_\mu\varphi,\varphi)$ is invariant under the continuous transformations {U}, L`=L with $\varphi_i$`$=U_i^j\varphi_j$ *for both free and interaction situations*. With that we have a principle to build the Lagrange and describe the associated particles. That is generated to the gauge invariance and gauge fields.

With that we have no problem with the changes on the masses, charges, spins,… , the particles are now classified by their symmetries, not by them. The method of calculation the changes on masses, charges, spins, … is the renormalization. One easy method is comparing the bare Lagrange $L_0(\partial_\mu\varphi_0,\varphi_0)$ and the bare fields φ_0 with the interaction Lagrange $L(\partial_\mu\varphi,\varphi)$ and interacted fields φ , the both in the same group representation, we consider the bare fields φ_0 as free and classical fields, so it is unchanged in the interactions and the associated bare masses, bare charges, bare spins,… are fixed values.

But to compare the interaction Lagrange with the bare one, the interaction results, like the self energy, …, must be finite(without divergences). For that we modify the propagator like eq2.2 for the photons and eq1.2 for the fermions, we set a→0 in the final results. But for the quarks we fix Z_2 and search for ka→0 so we can ignore the modifying and have the usual propagators. When ka>1 we can't ignore it, we give them a physical meaning, like to relate that modifying to Field dual behavior, free and composition, pairing particles-antiparticles appears as a scalar particle, we apply that for the quarks chapter 9 we find the static quarks interaction potential, we study the quarks plasma and use our results in Friedmann equations solutions.

The key words: Lagrange parameters, the chiral symmetry satisfying, nuclear potential, quarks static potential, fields dual behavior, quarks plasma phase, quarks condensation phase, the Big Bang, neither dark energy nor dark Matter.

1. The fermion self-energy in the electromagnetic interaction

We find the Lagrange parameters Z_2 and Z_m and make the results like the physical mass, using the modified propagators like 1.1 and let a →0 in the final results, as usual in the renormalization[1], [2]. But here we absorb the self energy to the Lagrange parameters Z_2 and Z_m(eq1.11). We do that for the electrons and generate it for the quarks. In the path integral of the electrons field we use the Lagrange

$L = i\bar\psi\gamma^\mu\partial_\mu\psi - m\bar\psi\psi + eA_\mu\bar\psi\gamma^\mu\psi$ the self energy becomes[2]

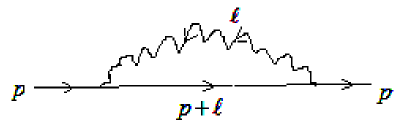


Diagram 1.1
electron self energy for the interaction with electromagnetic field

$$i\Sigma(\not P) = (ie)^2\frac{1}{i^2}\int\frac{d^4\ell}{(2\pi)^4}[\gamma^\mu\bar S(\not P + \not\ell)\gamma^\nu]\bar\Delta_{\mu\nu}(\ell^2) \quad \text{and} \quad \bar S(\not P) = \frac{-\not p + m}{p^2 + m^2 - i\varepsilon} \quad \textit{free fermion's propagator} \qquad 1.1$$

For photons propagator we modify:

$$\bar\Delta_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2 - i\varepsilon}\left(1 - \frac{a^2k^2}{1+a^2k^2}\right) = \frac{g_{\mu\nu}}{k^2-i\varepsilon}\frac{1}{1+\beta k^2} \quad : \beta = a^2 \qquad 1.2$$

The electron self-energy 1.1 becomes:

$$i\Sigma(\not P) = e^2\int\frac{d^4\ell}{(2\pi)^4}\frac{N}{(p+\ell)^2 + m^2}\frac{1}{\ell^2 + m_\gamma^2}\frac{1}{1+\beta\ell^2} \qquad 1.3$$

with $N = \gamma_\mu(-\not p - \not\ell + m)\gamma^\mu$

Using the Feynman formula:

$$\frac{1}{((p+\ell)^2 + m^2)\cdot(\ell^2 + m_\gamma^2)\cdot(1+\beta\ell^2)} = \int dF_3\frac{1}{\left[((p+\ell)^2 + m^2)x_1 + (\ell^2 + m_\gamma^2)x_2 + (1+\beta\ell^2)x_3\right]^3} \quad : \int dF_3 = 2\int_0^1 dx_1dx_2dx_3\delta(x_1+x_2+x_3-1) \qquad 1.4$$

and setting the transformation: $q=\ell+x_1p$

And changing the integral to be over q and making transformation to Euclidean space the electron self energy becomes

$$i\Sigma(\not P) = e^2 i\int\frac{d^4\bar q}{(2\pi)^4}\frac{1}{\beta}\int dF_3\frac{N}{[\,\bar q^2 + D\,]^3} \qquad 1.5$$

and D
$$D = -x_1^2p^2 + x_1p^2 + x_1m^2 + x_2m_\gamma^2 + (1-x_1-x_2)\frac{1}{\beta} \qquad 1.6$$

The linear term in q integrates to zero, using $q=1+x_1p$, N becomes[2]

$$N \to -2(1-x_1)\not p - 4m \qquad 1.7$$

Using the relation:
$$\int\frac{d^d\bar q}{(2\pi)^d}\frac{(\bar q^2)^a}{(\bar q^2 + D)^b} = \frac{\Gamma(b-a-\frac{d}{2})\Gamma(a+\frac{d}{2})}{(4\pi)^{\frac{d}{2}}\Gamma(b)\Gamma(\frac{d}{2})}D^{-(b-a-\frac{d}{2})}$$

The integral over q in Euclidean space is:

$$\Sigma(\not P) = e^2\frac{1}{\beta}\int dF_3 N\frac{\Gamma(3\text{-}2)\Gamma(2)}{(4\pi)^2\Gamma(3)\Gamma(2)}D^{-(3-2)} = e^2\frac{1}{\beta}\int dF_3\frac{N}{16\pi^2\cdot 2}D^{-1} \qquad 1.8$$

The self energy becomes

$$\Sigma(\not P) = e^2\frac{1}{\beta}\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{N}{16\pi^2}\frac{1}{D} = e^2\frac{1}{16\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{-2(1-x_1)\not p - 4m}{\beta\left[-x_1^2p^2 + x_1p^2 + x_1m^2 + x_2m_\gamma^2 + (1-x_1-x_2)\frac{1}{\beta}\right]} \qquad 1.9$$

$$=\frac{e^2}{8\pi^2}\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{-(1-x_1)\not{p}-2m}{[\beta f+(1-x_1-x_2)]} \quad with \quad f=-x_1^2p^2+x_1p^2+x_1m^2+x_2m_\gamma^2$$

Which is a finite result(without diverges). Now we renormalize the fermions propagator to give the real states and let a→0 .

The interacted electrons propagator becomes[2]:

$$\bar{S}(\not{P})^{-1}=\not{P}+m-\Sigma(\not{P}) \qquad 1.10$$

To renormalize the interacted field we rewrite: $\bar{S}(\not{P})^{-1}=\not{P}+m-\Sigma(\not{P})=Z_2\not{P}+Z_m m$ 1.11

The parameters Z_2 and Z_m are the renormalization parameters, later we try to make them constants like eq5.2 , for the interacted field Ψ we have:

$$\langle 0|\psi(\not{P})\bar{\psi}(-\not{P})\,|0\rangle=\frac{1}{i}\frac{1}{\not{P}+m-\Sigma(\not{P})}=\frac{1}{i}\frac{1}{Z_2\not{P}+Z_m m}=\frac{1}{Z_2 i}\frac{1}{\not{P}+Z_2^{-1}Z_m m}$$

We can rewrite $\langle 0|\sqrt{Z_2}\psi(\not{P})\sqrt{Z_2}\bar{\psi}(-\not{P})\,|0\rangle=\frac{1}{i}\frac{1}{\not{P}+Z_2^{-1}Z_m m}$

And make $m_0=Z_2^{-1}Z_m m \quad and \quad \psi_0=\sqrt{Z_2}\psi$ 1.12

With that we have bare fields Ψ_0 they are like the free fields and like the classical fields, so we can make them independent on the interaction, so $\frac{\partial\psi_0}{-\partial p^2}=0$ *and* $\frac{\partial m_0}{-\partial p^2}=0$ by that we renormalize the interaction. We make Ψ the interacted field with mass m the physical mass, but we have to make Re$\Sigma(-m)=0$ in 1.10 but with $(m_\gamma)^2<0$.

From 1.10 and 1.11 $Z_2=1+\frac{e^2}{8\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{1-x_1}{[\beta f+(1-x_1-x_2)]}$ 1.13

$$Z_m=1+\frac{e^2}{8\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{2}{[\beta f+(1-x_1-x_2)]} \quad : \quad f=-x_1^2p^2+x_1p^2+x_1m^2+x_2m_\gamma^2 \qquad 1.14$$

By that we removed the self energy of the interacted electron and make the mass varies.

we get the physical mass of the interacted electron, like usual, then let a→0 .

for easy we ignore the masses m and m_γ in f so $f\to -x_1^2p^2+x_1p^2$ therefore

$$\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{1-x_1}{[\beta f+(1-x_1-x_2)]}=\int_0^1 dx_1(1-x_1)\ln\left(1+\frac{1}{\mathrm{a}^2p^2x_1}\right)$$
$$=\frac{1}{2\left(\mathrm{a}^2p^2\right)^2}\left[-\mathrm{a}^2p^2+\ln(\mathrm{a}^2p^2+1)+2\mathrm{a}^2p^2\ln(\mathrm{a}^2p^2+1)+\left(\mathrm{a}^2p^2\right)^2\ln\left(1+1/\mathrm{a}^2p^2\right)\right]:\beta=\mathrm{a}^2$$

We have for x=a^{2}p^2→0: a→0

$$Z_2=1+\frac{\alpha}{4\pi}\left(\frac{3}{2}-\ln x+o(x)\right):\alpha=\frac{e^2}{4\pi}$$

For Z_m we have

$$\int_0^1 dx_1\ln\left(1+\frac{1}{\mathrm{a}^2p^2x_1}\right)=\frac{\ln\left(1+\mathrm{a}^2p^2\right)}{\mathrm{a}^2p^2}+\ln\left(\frac{1+\mathrm{a}^2p^2}{\mathrm{a}^2p^2}\right)$$

therefore Z_m becomes $Z_m=1+\frac{\alpha}{\pi}(1-\ln x+O(x))\ :\ x=\mathrm{a}^2p^2\to 0 \ when\ \mathrm{a}\to 0$

We run the mass m in $m_0 = Z_2^{-1} Z_m m$:

$$\ln(m_0) = -\ln(Z_2) + \ln(Z_m) + \ln(m):\ \frac{d}{dx}\ln(m_0) = 0$$

We consider only the first order in $\alpha = e^2/4\pi < 1$ we use $\ln(x+1) = x - x^2/2 + \ldots$

$$\ln Z_2 = \ln\left(1 + \frac{\alpha}{4\pi}\left(\frac{3}{2} - \ln(x)\right)\right) = \ln\left(1 + \frac{3\alpha}{8\pi} - \frac{\alpha}{4\pi}\ln(x)\right) = \ln\left((1 + 3\alpha/8\pi)(1 - \mathrm{b}_1 y)\right) = \ln(1 + 3\alpha/8\pi) - \mathrm{b}_1 y - \frac{1}{2}(\mathrm{b}_1 y)^2 + \ldots$$

$$\textit{with}\ \mathrm{b}_1 = \frac{\alpha/4\pi}{1 + 3\alpha/8\pi}\ \textit{and}\ y = \ln(x)$$

For Z_m

$$\ln Z_m = \ln\left(1 + \frac{\alpha}{\pi} - \frac{\alpha}{\pi}\ln(x)\right) = \ln\left((1 + \alpha/\pi)(1 - b_2 y)\right) = \ln(1 + \alpha/\pi) - b_2 y - \frac{1}{2}(b_2 y)^2 + \ldots$$

$$\textit{with}\ b_2 = \frac{\alpha/\pi}{1 + \alpha/\pi}$$

Therefore

$$\ln(m_0) = -\ln(1 + 3\alpha/8\pi) + \mathrm{b}_1 y + \frac{1}{2}(\mathrm{b}_1 y)^2 + \ldots + \ln(1 + \alpha/\pi) - b_2 y - \frac{1}{2}(b_2 y)^2 + \ldots + \ln(m)$$

$$\frac{d}{d\ln(x)} m_0 = \frac{d}{dy} m_0 = 0$$

That becomes

$$\ln(m_0) = -\ln(1 + 3\alpha/8\pi) + \mathrm{b}_1 y + \frac{1}{2}(\mathrm{b}_1 y)^2 + \ldots + \ln(1 + \alpha/\pi) - b_2 y - \frac{1}{2}(b_2 y)^2 + \ldots + \ln(m)$$

$$0 = \frac{3\alpha'/8\pi}{1 + 3\alpha/8\pi} + \mathrm{b}_1' y + \mathrm{b}_1 + (\mathrm{b}_1 y)(\mathrm{b}_1' y + \mathrm{b}_1) + \ldots + \frac{\alpha'/\pi}{1 + \alpha/\pi} - b_2' y - b_2 - (b_2 y)(b_2' y + b_2) + \ldots + m'/m$$

We consider only the first order in α , we will see $\alpha' \sim \alpha^2$, so $b' \sim \alpha^2$

$$\mathrm{b}_1 = \frac{\alpha/4\pi}{1 + 3\alpha/8\pi} \to \alpha/4\pi + o(\alpha^2)$$

Therefore

$$0 = \frac{3\alpha'/8\pi}{1 + 3\alpha/8\pi} + \mathrm{b}_1' y + \alpha/4\pi + O(\alpha^2) + (\mathrm{b}_1 y)(\mathrm{b}_1' y + \mathrm{b}_1) + \ldots + \frac{\alpha'/\pi}{1 + \alpha/\pi} - b_2' y - \alpha/\pi - O(\alpha^2) - (b_2 y)(b_2' y + b_2) + \ldots + m'/m$$

The first order $\alpha/4\pi - \alpha/\pi = -3\alpha/4\pi$ we write like

$$0 = \left[\frac{3\alpha'/8\pi}{1 + 3\alpha/8\pi} + \mathrm{b}_1' y + O(\alpha^2) + (\mathrm{b}_1 y)(\mathrm{b}_1' y + \mathrm{b}_1) + \ldots + \frac{\alpha'/\pi}{1 + \alpha/\pi} - b_2' y + O(\alpha^2) - (b_2 y)(b_2' y + b_2) + \ldots + \right] m - m\frac{3\alpha}{4\pi} + m'$$

We choice the solution $-m\frac{3\alpha}{4\pi} + m' = \frac{-m}{\ldots + k_{-1} y^{-1} + k_0 + k_1 y + k_2 y^2 + \ldots} \to 0:\ y = \ln(\mathrm{a}^2 p^2) \to \infty\ \textit{when}\ \mathrm{a} \to 0$

The constants k_i can be determined to satisfy

$$0=\left[\frac{3\alpha'/8\pi}{1+3\alpha/8\pi}+\mathrm{b}_1'y+O(\alpha^2)+(\mathrm{b}_1y)(\mathrm{b}_1'y+\mathrm{b}_1)+...+\frac{\alpha'/\pi}{1+\alpha/\pi}-b_2'y+O(\alpha^2)-(b_2y)(b_2'y+b_2)+...\right]-\frac{1}{\left(..+k_{-1}y^{-1}+k_0+k_1y+...\right)}$$

so

$$\left[\frac{3\alpha'/8\pi}{1+3\alpha/8\pi}+\mathrm{b}_1'y+O(\alpha^2)+(\mathrm{b}_1y)(\mathrm{b}_1'y+\mathrm{b}_1)+...+\frac{\alpha'/\pi}{1+\alpha/\pi}-b_2'y-(b_2y)(b_2'y+b_2)+...\right]\left(...+k_{-1}y^{-1}+k_0+k_1y+...\right)=1$$

Running the mass

$$-m\frac{3\alpha}{4\pi}+m'=0\rightarrow\frac{1}{m}\frac{dm}{d\ln(\mathrm{a}^2p^2)}=\frac{1}{m}\frac{dm}{d\ln(-p^2)}:\ \mathrm{a}=\text{constant}\rightarrow 0$$
$$\rightarrow\frac{1}{m}\frac{dm}{d\ln(-p^2)}-\frac{3\alpha}{4\pi}=0\rightarrow\frac{1}{m}\frac{dm}{d\alpha}\frac{d\alpha}{d\ln(-p^2)}-\frac{3\alpha}{4\pi}=0\rightarrow\frac{1}{m}\frac{dm}{d\alpha}\beta(\alpha)-\frac{3\alpha}{4\pi}=0$$

So we cancelled the self energy and made the mass carries the energy, with that, the interacted electrons are like free particles, but the mass varies.

We need to find Z_2 and Z_m for the quarks in the interaction with the gluons(strong interaction), in *SU(3)* representation, with that, eq1.3 becomes, for arbitrary *SU(N)*:

$$i\Sigma_{ij}(\not{P})=\int\frac{d^4\ell}{(2\pi)^4}\frac{g_s(T^a)_i^k\gamma_\mu(-\not{p}-\not{\ell}+m)\delta_{kl}\,g_s(T^b)_j^l\gamma^\mu}{(p+\ell)^2+m^2}\frac{\delta^{ab}}{\ell^2+\mathrm{m}_\gamma^2}\frac{1}{1+\beta\ell^2}$$

$$\text{and}\quad\sum(T^a)_i^k\delta_{kl}(T^b)_j^l\delta^{ab}=\sum(T^a)_{il}(T^a)_j^l=\sum_a(T^aT^a)_{ij}=C(R)\delta_{ij}\qquad 1.17$$

Therefor the parameters Z_2 and Z_m becomes

$$Z_2=1-C(R)\frac{g^2}{8\pi^2}\int_0^1dx_1\int_0^{1-x_1}dx_2\frac{1-x_1}{[\beta f+(1-x_1-x_2)]}\qquad 1.18$$
$$Z_m=1-C(R)\frac{g^2}{8\pi^2}\int_0^1dx_1\int_0^{1-x_1}dx_2\frac{2}{[\beta f+(1-x_1-x_2)]}\quad:\quad f=-x_1^2p^2+x_1p^2+x_1m^2+x_2m_\gamma^2$$

And the factor $C(R)=(N^2-1)/2N$, for the quarks N=3 .

Notice: the logarithm series $\ln(x+1)=x-x^2/2+x^3/3+\ldots$ is satisfied for any x:

$$\ln(x+1)=\int\frac{dx}{x+1}\ \ for\ any\ x$$

And the series

$$(x+1)\left(1-x+x^2-x^3+...\right)=1\ is\ satisfied\ for\ any\ x\ so\ \frac{1}{x+1}=1-x+x^2-x^3+...$$

Therefore $\ln(x+1)=\int\frac{dx}{x+1}=x-\frac{x^2}{2}+\frac{x^3}{3}+...$ is satisfied for any x .

2. Photon self-energy in spinor electrodynamics interaction

We find the Lagrange parameter Z_3 and make the result, the coupling constant α as in the usual Renormalizations. But here we absorb the self energy of the photons to the Lagrange parameters Z_3, in the path integral of the Photons field, the self-energy is given by[2]

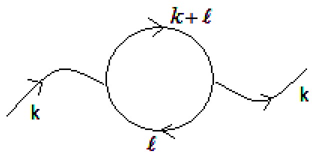


Diagram 2.1
Photon self energy in spinor electrodynamics interaction

$$i\Pi^{\mu\nu}(k^2) = -(ie)^2 \frac{1}{i^2}\int \frac{d^4\ell}{(2\pi)^4} \mathrm{Tr}[\bar{S}(\not{k}+\not{\ell})\gamma^\mu \bar{S}(\not{\ell})\gamma^\nu] + \cdots \quad with\ L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + eA_\mu\bar{\psi}\gamma^\mu\psi \qquad 2.1$$

To calculate it with a result without diverges we use the modifying:

$$\bar{S}(p) = \frac{-\not{p}+m}{p^2+\mathrm{m}^2-i\varepsilon}\cdot\frac{1}{1+\beta p^2} \qquad 2.2$$

We find:

$$i\,\Pi^{\mu\nu}(k^2) = -e^2\int\frac{d^4\ell}{(2\pi)^4}\frac{\mathrm{Tr}[(-\not{k}-\not{\ell}+m)\gamma^\mu(-\not{\ell}+m)\gamma^\nu]}{(1+\beta(\ell+k)^2)\cdot((k+\ell)^2+m^2)\cdot(\ell^2+m^2)\cdot(1+\beta\ell^2)} \qquad 2.3$$

Using $\frac{1}{(1+\beta(\ell+k)^2)\cdot((k+\ell)^2+m^2)\cdot(\ell^2+m^2)\cdot(1+\beta\ell^2)} = \frac{1}{\beta^2}\int dF_4\left[(\frac{1}{\beta}+(\ell+k)^2)x_1+((k+\ell)^2+m^2)x_2+(\ell^2+m^2)x_3+(\frac{1}{\beta}+\ell^2)x_4\right]^{-4}$

with $\int dF_4 = 6\int_0^1 dx_1dx_2dx_3dx_4\delta(x_1+x_2+x_3+x_4-1)$

The self energy becomes

$$i\,\Pi^{\mu\nu}(k^2) = -e^2\frac{1}{\beta^2}\int\frac{d^4\ell}{(2\pi)^4}\int dF_4\frac{4N^{\mu\nu}}{\left[(\frac{1}{\beta}+(\ell+k)^2)x_1+((k+\ell)^2+m^2)x_2+(\ell^2+m^2)x_3+(\frac{1}{\beta}+\ell^2)x_4\right]^4} \qquad 2.4$$

Where $4N^{\mu\nu}$ is: $\quad 4N^{\mu\nu} = \mathrm{Tr}\,[(-\not{k}-\not{\ell}+m)\gamma^\mu(-\not{\ell}+m)\gamma^\nu]$

completing the trace[2], we have: $N^{\mu\nu} = (k+\ell)^\mu\ell^\nu + (k+\ell)^\nu\ell^\mu - \left((k+\ell)\ell+m^2\right)\mathrm{g}^{\mu\nu}$

We set the translation: $q = \ell + (x_1+x_2)k$

Changing the integral to be over q and making transformation to Euclidean space. And dropping the terms linear in q(because they integrate to zero)

$$N^{\mu\nu} \to 2q^\mu q^\nu - 2(x_1+x_2)(1-x_1-x_2)k^\mu k^\nu - \left(q^2-(x_1+x_2)(1-x_1-x_2)k^2+m^2\right)\mathrm{g}^{\mu\nu} \qquad 2.5$$

Using the relation :?$\int d^dq^\mu q^\nu f\ q^2 = \frac{1}{d}\ {}^{\mu\nu}\int d^dq^2 f\ q^2$ which allow us to replace $q^\mu q^\nu$ in $N^{\mu\nu}$ like:

$$N^{\mu\nu} \to -2(x_1+x_2)(1-x_1-x_2)k^\mu k^\nu + \left(-\frac{1}{2}q^2+(x_1+x_2)(1-x_1-x_2)k^2-m^2\right)\mathrm{g}^{\mu\nu} \qquad 2.6$$

It becomes $\quad i\,\Pi^{\mu\nu}(k^2) = -e^2\frac{1}{\beta^2}\int\frac{d^4\bar{q}}{(2\pi)^4}\int dF_4\frac{4N^{\mu\nu}}{\left[\bar{q}^2+D\right]^4} \qquad 2.7$

With $\quad D = -(x_1+x_2)^2k^2 + (x_1+x_2)k^2 + (x_2+x_3)m^2 + \frac{1}{\beta}(1-x_2-x_3)$

Using the relation: $\int\frac{d^d\bar{q}}{(2\pi)^d}\frac{(\bar{q}^2)^a}{(\bar{q}^2+D)^\mathrm{b}} = \frac{\Gamma(\mathrm{b}-a-\frac{d}{2})\Gamma(a+\frac{d}{2})}{(4\pi)^{\frac{d}{2}}\Gamma(\mathrm{b})\Gamma(\frac{d}{2})}D^{-(\mathrm{b}-a-\frac{d}{2})}$

Integrate over q in the Euclidean space, the photon self-energy becomes:

$$i\,\Pi^{\mu\nu}(k^2)=-\frac{e^2}{2\pi^2}i\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{(x_1+x_2)(1-x_1-x_2)(k^2g^{\mu\nu}-k^\mu k^\nu)}{[\beta f+(1-x_2-x_3)]^2}-\frac{e^2}{4\pi^2}i\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{(1-x_2-x_3)}{[\beta f+(1-x_2-x_3)]^2}\left(\frac{1}{\beta}-m^2\right):2.8$$

$$with\quad f=-(x_1+x_2)^2k^2+(x_1+x_2)k^2+(x_2+x_3)m^2$$

We have a problem in 2.3 , when $\beta\to 0$ the integrals diverge like

$$\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{(x_1+x_2)(1-x_1-x_2)(k^2g^{\mu\nu}-k^\mu k^\nu)}{(1-x_1-x_2-x_3)^2}$$

Therefore we rewrite it like

$$i\,\Pi^{\mu\nu}(k^2)=-\frac{e^2}{2\pi^2}i\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{(x_1+x_2)(1-x_1-x_2)(k^2g^{\mu\nu}-k^\mu k^\nu)}{[\beta f+(1-x_1-x_2-x_3)]^2}$$

So

$$\Pi^{\mu\nu}(k^2)=-\frac{e^2}{2\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{(x_1+x_2)(1-x_1-x_2)k^2P^{\mu\nu}}{[\beta f+(1-x_1-x_2-x_3)]^2}\qquad 2.9$$

The photon self-energy can be written like[2] $\Pi^{\mu\nu}(k^2)=k^2\Pi(k^2)\,P^{\mu\nu}(k)$ with the perturbed photons propagator : $\bar{\Delta}_{\mu\nu}(k^2)=\frac{\mathrm{P}_{\mu\nu}(k)}{k^2[1-\Pi(k^2)]-i\varepsilon}$ and the projection operator : $\mathrm{P}^{\mu\nu}(k)=g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}$

Using that, we find: $$\Pi(k^2)=-\frac{e^2}{2\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{(x_1+x_2)(1-x_1-x_2)}{[\beta f+(1-x_1-x_2-x_3)]^2}\qquad 2.10$$

We define the parameter Z_3 for the quantum electromagnetic Field A^μ via: $Z_3=1-\Pi(k^2)$ Therefore the interacted photon propagator becomes:

$$\bar{\Delta}_{\mu\nu}(k^2)=\frac{\mathrm{P}_{\mu\nu}(k)}{k^2Z_3-i\varepsilon}=\frac{1}{Z_3}\frac{\mathrm{P}_{\mu\nu}(k)}{k^2-i\varepsilon}\qquad 2.11$$

We renormalize the interacted field A_μ like 1.12: $\mathrm{A}_0^\mu=\sqrt{Z_3}\mathrm{A}^\mu$? 2.12

The parameter Z_3 is finite: $Z_3=1+\frac{e^2}{2\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{(x_1+x_2)(1-x_1-x_2)}{[\beta f+(1-x_1-x_2-x_3)]^2}$

It becomes, ignoring the mass *m* and m_γ in *f*

$$Z_3=1+\frac{e^2}{2\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{(x_1+x_2)(1-x_1-x_2)}{[\beta f+(1-x_1-x_2-x_3)]^2}=1+\frac{e^2}{2\pi^2}\left[\frac{1}{x^2}-\frac{\ln(x+1)}{x^3}\right]\;:\;x=\mathrm{a}^2k^2\qquad ?\qquad 2.13$$

We need to write it in ln(x) terms so we expand it near $x=x_0$, then we make $x_0\to 0$, we use

$$f(x)=f(x_0)+x\frac{\partial f(x)}{\partial x}\bigg|_{x=x_0}\frac{\ln(x/x_0)}{1!}+\ldots\quad or\quad f(x)=\sum_{n=0}^{\infty}c_n\left(\ln(x/x_0)\right)^n\ with\ c_n=\frac{1}{n!}\left(x\frac{\partial}{\partial x}\right)^n f(x)\bigg|_{x=x_0}$$

2.13 becomes, setting $y=\ln(x/x_0)$ and using $\ln(x+1)=x-x^2/2+x^3/3+\ldots$

$$Z_3=c+by+o(y^2)\quad with\quad c=1+\frac{2\alpha}{\pi}\left[\frac{1}{x_0^2}-\frac{\ln(x_0+1)}{x_0^3}\right]\quad\to 1+\frac{2\alpha}{\pi}\left(\frac{1}{2x_0}-\frac{1}{3}+O(x_0)\right)\;:\;x_0\to 0$$

$$and\quad b=\frac{2\alpha}{\pi}x\frac{d}{dx}\left[\frac{1}{x^2}-\frac{\ln(x+1)}{x^3}\right]\bigg|_{x=x_0}\to\frac{2\alpha}{\pi}\left(-\frac{1}{x_0^2(x_0+1)}+\frac{1}{x_0^2}-\frac{3}{2x_0}+1+O(x_0)\right)\quad ?$$

We renormalize the interacted Field A_μ like $\mathrm{A}_0^\mu=\sqrt{Z_3}\mathrm{A}^\mu$

from the interaction term $A_\mu \bar{\psi} V^\mu \psi$ with $V^\mu = eZ_1\gamma^\mu$ (eq3.6), we can write

$$eA_\mu \bar{\psi} V^\mu \psi \to \frac{eZ_1}{Z_2\sqrt{Z_3}}\sqrt{Z_3}A_\mu \sqrt{Z_2}\bar{\psi}\gamma^\mu \sqrt{Z_2}\psi \;\to\; \frac{eZ_1}{Z_2\sqrt{Z_3}}A_{0\mu}\bar{\psi}_0\gamma^\mu\psi_0 = e_0 A_{0\mu}\bar{\psi}_0\gamma^\mu\psi_0$$

We make the bare fields like classical fields, so e_0 is constant. From the relation(which is related to the Ward identify[2]): $(p'-p)_\mu \mathrm{V}^\mu(p',p) = e\left[\bar{S}(\not{P}')^{-1} - \bar{S}(\not{P})^{-1}\right]$

We use it for the interacted field Ψ so we use 1.11 and 3.6 :

$$(p'-p)_\mu eZ_1\gamma^\mu = eZ_2\left[\not{P}' - \not{P}\right] = eZ_2(p'-p)_\mu\gamma^\mu$$

It must be $Z_2 = Z_1$ for gauge invariance[2], we have $e_0 = \frac{e}{\sqrt{Z_3}}$, $\alpha_0 = \frac{e_0^2}{4\pi} = \frac{\alpha}{Z_3}$ therefore

$$\frac{d}{dy}\ln(\alpha_0) = 0 \to \frac{d}{dy}\ln(\alpha) - \frac{d}{dy}\ln(Z_3) = 0 \to \frac{\alpha'}{\alpha} - \frac{d}{dy}\ln\left(c + by + O(y^2)\right) = 0 \;\; with\; y = \ln x = \ln(\mathrm{a}^2k^2)$$

we write $$\ln c\left(1 + \frac{b}{c}y + O(y^2)\right) = \ln c + \ln\left(1 + \frac{b}{c}y + O(y^2)\right) \; and \; \ln\left(1 + \frac{b}{c}y + O(y^2)\right) = \frac{b}{c}y + O(y^2)$$

$$\frac{\alpha'}{\alpha} - \frac{c'}{c} - \frac{b}{c} + O(y) = 0$$

That becomes in y=0: $x = x_0 \to 0$

$$-\frac{c'}{c} - \frac{b}{c} + \frac{\alpha'}{\alpha} = 0 \to -c' - b + c\frac{\alpha'}{\alpha} = 0$$

$$\to -\frac{2\alpha'}{\pi}\left(\frac{1}{2x_0} - \frac{1}{3} + O(x_0)\right) - \frac{2\alpha}{\pi}\left(-\frac{1}{x_0^2(x_0+1)} + \frac{1}{x_0^2} - \frac{3}{2x_0} + 1 + O(x_0)\right) + \left(1 + \frac{2\alpha}{\pi}\left(\frac{1}{2x_0} - \frac{1}{3} + O(x_0)\right)\right)\frac{\alpha'}{\alpha} = 0$$

reorder it like

$$\frac{\alpha'}{\alpha}\left(1 - \frac{2\alpha}{3\pi}\right) + \frac{2\alpha'}{3\pi} - \frac{2\alpha}{\pi} + \frac{2\alpha}{\pi}\left(\frac{1}{2x_0} + O(x_0)\right)\frac{\alpha'}{\alpha} - \frac{2\alpha}{\pi}\left(-\frac{1}{x_0^2(x_0+1)} + \frac{1}{x_0^2} - \frac{3}{2x_0} + O(x_0)\right) - \frac{2\alpha'}{\pi}\left(\frac{1}{2x_0} + O(x_0)\right) = 0$$

We can assume

$$\frac{\alpha'}{\alpha}\left(1 - \frac{2\alpha}{3\pi}\right) + \frac{2\alpha'}{3\pi} - \frac{2\alpha}{\pi} = \frac{-1}{\ldots + k_{-1}x_0^{-1} + k_0 + k_1x_0 + \ldots} \to 0 :\; x_0 \to 0$$

The parameters k_i can be determined to satisfy

$$\frac{-1}{\ldots + k_{-1}x_0^{-1} + k_0 + k_1x_0 + \ldots} + \frac{2\alpha}{\pi}\left(\frac{1}{2x_0} + O(x_0)\right)\frac{\alpha'}{\alpha} - \frac{2\alpha}{\pi}\left(-\frac{1}{x_0^2(x_0+1)} + \frac{1}{x_0^2} - \frac{3}{2x_0} + O(x_0)\right) - \frac{2\alpha'}{\pi}\left(\frac{1}{2x_0} + O(x_0)\right) = 0$$

so

$$\left(\ldots + k_{-1}x_0^{-1} + k_0 + k_1x_0 + \ldots\right)\left[\frac{2\alpha}{\pi}\left(\frac{1}{2x_0} + O(x_0)\right)\frac{\alpha'}{\alpha} - \frac{2\alpha}{\pi}\left(-\frac{1}{x_0^2(x_0+1)} + \frac{1}{x_0^2} - \frac{3}{2x_0} + O(x_0)\right) - \frac{2\alpha'}{\pi}\left(\frac{1}{2x_0} + O(x_0)\right)\right] = 1$$

and

$$\frac{\alpha'}{\alpha} - \frac{2\alpha}{\pi} = 0 \to \frac{d\alpha}{dy} = \frac{2\alpha^2}{\pi} = \frac{d\alpha}{d\ln\left(x/x_0\right)} = \frac{d\alpha}{d\ln\left(-k^2\right)} = \beta(\alpha) :\; x = \mathrm{a}^2p^2 \text{ and a=constant} \to 0$$

$\beta(\alpha)$ is the beta function, the solution(the electromagnetic coupling constant)

$$\alpha_e = \frac{c}{-\ln\left(\frac{-k^2}{\Lambda}\right)} : -k^2 < \Lambda$$

With that we removed the photons self energy, so the interacted photons are like free particles, while the coupling constant α_e depends on the energy.

3. The fermion –photon –vertex

Like what we did before we find the Lagrange parameter Z_1(p', p) using the modified propagators and let a→0 .

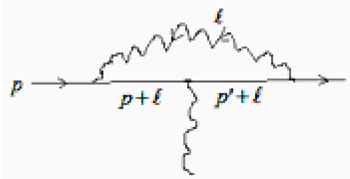


Diagram 3.1
The fermion-photon vertex

$$iV^\mu(p',p) = ie\gamma^\mu + iV^\mu(p',p)_{1-loop} + O(e^5) \qquad 3.1$$

$$with \;\; iV^\mu(p',p)_{1-loop} = (ie)^3\frac{1}{i^3}\int\frac{d^4\ell}{(2\pi)^4}[\gamma^\rho\bar{S}(\not{p}' + \not{\ell})\gamma^\mu\bar{S}(\not{p} + \not{\ell})\gamma^\nu]\bar{\Delta}_{\nu\rho}(\ell^2)$$

$$= e^3\int\frac{d^4\ell}{(2\pi)^4}[\gamma^\rho\bar{S}(\not{p}' + \not{\ell})\gamma^\mu\bar{S}(\not{p} + \not{\ell})\gamma^\nu]\bar{\Delta}_{\nu\rho}(\ell^2) \qquad 3.2$$

We use the modifying

$$\bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2+\beta k^4} = \frac{1}{k^2}\cdot\frac{g_{\mu\nu}}{1+\beta k^2} \qquad \text{Adding a mass}: \bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2+\beta k^4} \to \frac{g_{\mu\nu}}{k^2+m_\gamma^2}\cdot\frac{1}{1+\beta k^2} \qquad 3.3$$

The vertex becomes:

$$iV^\mu(p',p)_{1-loop} = e^3\int\frac{d^4\ell}{(2\pi)^4}\frac{N^\mu}{((\ell+p')^2+m^2)\cdot((p+\ell)^2+m^2)\cdot(\ell^2+m_\gamma^2)\cdot(1+\beta\ell^2)}$$

$$= e^3\frac{1}{\beta}\int\frac{d^4\ell}{(2\pi)^4}\frac{N^\mu}{((\ell+p')^2+m^2)\cdot((p+\ell)^2+m^2)\cdot(\ell^2+m_\gamma^2)\cdot(\frac{1}{\beta}+\ell^2)} \qquad 3.4$$

$$with \qquad N^\mu = \gamma_\nu(-\not{p}' - \not{\ell} + m)\gamma^\mu(-\not{p} - \not{\ell} + m)\gamma^\nu$$

Using the Feynman formula

$$\frac{1}{((\ell+p')^2+m^2)\cdot((p+\ell)^2+m^2)\cdot(\ell^2+m_\gamma^2)\cdot(\frac{1}{\beta}+\ell^2)} = \int dF_4\left[((\ell+p')^2+m^2)x_1 + ((p+\ell)^2+m^2)x_2 + (\ell^2+m_\gamma^2)x_3 + (\frac{1}{\beta}+\ell^2)x_4\right]^{-4}$$

$$with \;\; \int dF_4 = 6\int_0^1 dx_1 dx_2 dx_3 dx_4\,\delta(x_1+x_2+x_3+x_4-1)$$

setting the change q=ℓ+x_1p'+x_2p And let the integral over q and making a transformation to the Euclidean space the vertex becomes:

$$iV^\mu(p',p)_{1-loop} = e^3\frac{1}{\beta}i\int\frac{d^4\bar{q}}{(2\pi)^4}\int dF_4\frac{N^\mu}{(\bar{q}^2+D)^4}$$

And D: $D=-x_1^2p'^2+x_1p'^2-x_2^2p^2+x_2p^2+(x_1+x_2)m^2-2x_1x_2p\cdot p'+x_3m_\gamma^2+(1-x_1-x_2-x_3)\frac{1}{\beta}$

using $q=\ell+x_1p'+x_2p$ to omit ℓ , N^μ becomes[2]:

$$N^\mu=\gamma_\nu(-\not{q}+x_1\not{p}'-(1-x_2)\not{p}+m)\gamma^\mu(-\not{q}-(1-x_1)\not{p}'+x_2\not{p}+m)\gamma^\nu=\gamma_\nu\not{q}\gamma^\mu\not{q}\gamma^\nu+\bar{N}^\mu+\textit{liner term}\ \text{in}\ q$$

$$\textit{with}\quad \bar{N}^\mu=\gamma_\nu\left[x_1\not{p}'-(1-x_2)\not{p}+m\right]\gamma^\mu\left[-(1-x_1)\not{p}'+x_2\not{p}+m\right]\gamma^\nu$$

using gamma matrices properties $\gamma_\nu\not{q}\,\gamma^\mu\not{q}\,\gamma^\nu\to q^2\,\gamma^\mu$ and dropping the linear terms in q because they integrate to zero, the vertex becomes

$$\mathrm{V}^\mu(p',p)_{1-loop}=e^3\frac{1}{\beta}\int\frac{d^4\bar{q}}{(2\pi)^4}\int dF_4\frac{1}{\left(\bar{q}^2+D\right)^4}\left(\bar{q}^2\gamma^\mu+\bar{\mathrm{N}}^\mu\right)$$

For renormalization the vertex $e\gamma^\mu$ we consider only the term $q^2\gamma^\mu$:

$$\mathrm{V}^\mu(p',p)_{1-loop}=e^3\gamma^\mu\frac{1}{\beta}\int dF_4\frac{\Gamma(4-1-2)\Gamma(1+2)}{(4\pi)^2\Gamma(4)\Gamma(2)}D^{-(4-1-2)}=e^3\gamma^\mu\frac{1}{\beta}\int dF_4\frac{\Gamma(4-1-2)\Gamma(1+2)}{(4\pi)^2\Gamma(4)\Gamma(2)}D^{-(4-1-2)}$$

$$=e^3\gamma^\mu\int dF_4\frac{1}{3\cdot(4\pi)^2}\frac{1}{\beta D} \qquad 3.5$$

$\textit{with}\ \ \beta D=\beta f+(1-x_1-x_2-x_3)$

We see, the vertex V^μ (p’, p) is finite, does not diverge.

$$\mathrm{V}^\mu(p',p)_{total}=e\gamma^\mu\left[1+\frac{e^2}{8\pi^2}\int_0^1dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{1}{\beta f+(1-x_1-x_2-x_3)}\right]=e\gamma^\mu z_1 \qquad 3.6$$

Z_1 is the vertex parameter(renormalization parameter)

$$Z_1=1+\frac{e^2}{8\pi^2}\int_0^1dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_1-x_2}dx_3\frac{1}{\beta f+(1-x_1-x_2-x_3)}\ :\ \beta=\mathrm{a}^2\to0 \qquad 3.7$$

In our next study we try to make Z_2 and Z_m constants by running the length a and search for the condition a→0 . we can’t make Z_3 constant, because we need it to run the coupling constant and have a choice to make Z_2 , Z_1 and Z_m constants.

Z_2 is fixed→ the length a depends on the fermion energy *p* .

Z_1 is fixed→ the length a depends on the photon transferred energy *q=p’-p* .

4. Chiral symmetry

The chiral symmetry is the symmetry of the massless fermions Lagrange. Which is the Symmetry $U_L \neq U_R$ here, we distinguish between the chiral symmetry which associates with the gauge invariance and the chiral symmetry which associates with the flavor invariance(like the quarks). We assume that they aren't conserved separately, but together they are. We see that in quarks dual behavior, chapter 9, Feynman diagrams.

In this chapter we find $\partial_\mu J^{\mu5}=0$ for SU(1) Symmetry using the modified fields propagators(particles dual behavior: singles and pairing particle-antiparticle), with that, the gauge invariance and flavor invariance together satisfy the chiral symmetry under the quantum fluctuation. That appears in the quarks interaction diagrams chapter 9 these diagrams satisfy the chiral symmetry for gauge invariance SU(1), also contain π-particles which satisfy the flavor invariance.

In this chapter we verify $\partial_\mu J^{\mu5}=0$ for SU(1) invariance.

We assume that the chiral symmetry is not satisfied because of the polarized quantum fluctuation due to the existence of the separated charges, but if the chiral symmetry is associated with more stationary states, such $U(r)_{r\to 0}$ is finite, we can believe in it.

from the relation :

$$\partial_\mu J^\mu_A = -\frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu}F_{\rho\sigma}$$

we find it is trivial to say $\partial_\mu J^{\mu 5}\neq 0$ when the Field A^μ in the last equation oscillates(quantum Field), because the strength $F_{\mu\nu}$ also oscillates, so $\partial_\mu J^{\mu 5}$ oscillates and takes the zero value alternately($J^{\mu 5}$ is alternately conserved). But if the field A_μ is fixed (classical Field separates the charges), we can say $\partial_\mu J^{\mu 5}\neq 0$, but there will be a vacuum polarization.

we remove the vacuum polarization using the modified propagators like eq1.2 the conservation $\partial_\mu J^{\mu 5}=0$ is never seen at high distances, because as we said the space would be polarized. But it is seen at low distances $\Delta r\to 0$ where the gauge forces become linear.

Using the amplitude 4.4 for the axial current $J^{\mu 5}(x)$ to create two photons, we have:

$$\partial_\mu J^{\mu 5} = -\frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu}F_{\rho\sigma}$$

The problem which leads to $\partial_\mu J^{\mu 5}\neq 0$ is the divergence in the integral 4.20, but by using the modifying

$$\bar{S}(p) = \frac{-\not p}{p^2 - i\varepsilon}\frac{1}{1+a^2p^2} \quad , \quad \bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2 - i\varepsilon}\frac{1}{1+a^2k^2}$$

the integral 4.20 would converge, and we would have hidden chiral symmetry $\partial_\mu J^{\mu 5}=0$.

the massless fermions Lagrange: $L_0 = i\bar\psi\not\partial\psi$ 4.1

with it the chiral symmetry is classically satisfied $\partial_\mu J^{\mu 5} = 0$, $J^{5} = \bar\psi\gamma^\mu\gamma^5\psi$ 4.2

but we don't have any term in the Lagrange L_0 includes the Axial current $J^{\mu 5}$, so we do not know its effects on the quantum processes, or it is not visible, so its effect is hidden .

The amplitude for the axial current $J^{\mu 5}(x)$ to create two photons is[2]:

$$\langle p,q|\, j^{\mu 5}(x)|0\rangle \qquad 4.3$$

Where p and q are the momentums of the two created photons .

But we have a problem, this is, we can't insert the axial current $J^{\mu 5}$ in the Lagrange as done for the vector current J^μ which is included in the Lagrange, such $A_\mu J^\mu$.

We can solve that problem by assuming that for the flavor symmetry, the field dual behavior gives scalar charged particles like the pions(Feynman diagrams, chapter 9), with them the Axial current interact indirectly with the electromagnetic field.

The amplitude 4.3 can be wrote like[2]:

$$\langle p,q|\, j^\rho_A(z)|0\rangle = -e^2\varepsilon_{?}\varepsilon'_\nu C^{\mu\nu\rho}(p,\, q,\, r)e^{irz}\Big|_{r=-p-q} \qquad 4.4$$

Where ε_μ , ε'_ν are polarization vectors of the two produced photons. Using the LSZ formula, $C^{\mu\nu\rho}(p,q,r)$ is:

$$(2\pi)^4\delta^4(p+q+r)C^{\mu\nu\rho}(p,\, q,\, r) \doteqdot \int d^4x\, d^4y\, d^4z\, e^{-i(px+qy+rz)}\langle 0|Tj^\mu(x)\,j^\nu(y)\,j^\rho_A(z)|0\rangle \qquad 4.5$$

Now we want to know where the problem $\partial_\mu J^{\mu 5}\neq 0$ comes from ?

We begin from the relation[2]

$$\langle p,q|\, j_A^{\rho}(z)|0\rangle \;=\; -e^2\varepsilon_{?}\varepsilon'_{\nu}\, C^{\mu\nu\rho}(p,\, q,\, r)e^{irz}\Big|_{r=-p-q} \qquad 4.7$$

Using the relation $(2\pi)^4\delta^4(p+q+r)C^{\mu\nu\rho}(p,\, q,\, r)?=\int\ ^4\ ^4\ ^4\ e^{-i(px+qy+rz)}\langle\ |\ ^{\mu}(\ )\ ^{\nu}(\ )\ ^{\rho}_{A}(\ )|\ \rangle$

we get :

$$iV^{\mu\nu\rho}(p,\, q,\, r) \;=\; -\frac{1}{2}(ig)^3 C^{\mu\nu\rho}(p,\, q,\, r) + O(g5) \qquad 4.8$$

with:

$$iV^{\mu\nu\rho}(p,\, q,\, r) \;=\; (-1)(ig)^3\left(\frac{1}{i}\right)^3\int\frac{d^4\ell}{(2\pi)^4}\frac{N^{\mu\nu\rho}}{(\ell-p)^2\,\ell^2(\ell+q)^2} \qquad 4.9$$
$$+\;(p,\, \mu\leftrightarrow q,\, \nu)\;+\;O(g5)$$

and $N^{\mu\nu\rho}$:

$$N^{\mu\nu\rho} = \frac{1}{2}Tr[(-\not{\ell}+\not{p})\gamma\ (-\not{\ell})\gamma^{\nu}(-\not{\ell}-\not{q})\gamma^{\rho}\gamma_5] \qquad 4.10$$

Using the trace circle property we find:

$$N^{\mu\nu\rho} = \frac{1}{2}Tr[(\not{\ell}-\not{p})\gamma\ \not{\ell}\gamma^{\nu}(\not{\ell}+\not{q})\gamma^{\rho}\gamma_5] \qquad 411$$

Taking derivative the relation:

$$\langle p,q|\, j_A^{\rho}(z)|0\rangle \;=\; -e^2\varepsilon_{?}\varepsilon'_{\nu}\, C^{\mu\nu\rho}(p,\, q,\, r)e^{irz}\Big|_{r=-p-q}$$

we get[2]

$$\langle p,q|\partial_{\rho}j_A^{\rho}(z)|0\rangle \;=\; -ie^2\varepsilon_{?}\varepsilon'_{\nu}\, r_{\rho}C^{\mu\nu\rho}(p,\, q,\, r)e^{irz}\Big|_{r=-p-q} \qquad 4.12$$

From

$$iV^{\mu\nu\rho}(p,\, q,\, r) = -\frac{1}{2}(ig)^3 C^{\mu\nu\rho}(p,\, q,\, r) + O(g^5)$$

we have

$$ir_{\rho}V^{\mu\nu\rho}(p,\, q,\, r) \;=\; -\frac{1}{2}(ig)^3 r_{\rho}C^{\ \nu\rho}(p,\, q,\, r) \;+\; O(g5) \qquad 4.13$$

We find $r_{\rho}C^{\mu\nu\rho}$ using the relation

$$iV^{\mu\nu\rho}(p,\, q,\, r) \;=\; (-1)(ig)^3\left(\frac{1}{i}\right)^3\int\frac{d^4\ell}{(2\pi)^4}\frac{N^{\mu\nu\rho}}{(\ell-p)^2\,\ell^2(\ell+q)^2} \;+\;(p,\, \mu\leftrightarrow q,\, \nu)\;+\;O(g^5)$$

We get:

$$r_{\rho}V^{\mu\nu\rho}(p,\, q,\, r) \;=\; ig^3\int\frac{d^4\ell}{(2\pi)^4}?\frac{r_{\rho}N^{\mu\nu\rho}}{(\ell-p)^2\,\ell^2(\ell+q)^2?}+(p,\, ?\ \leftrightarrow\ ,\ \nu)q +\ (\ ^5O\ g \qquad 4.14$$

From the

$$N^{\mu\nu\rho} = \frac{1}{2}Tr[(\not{\ell}-\not{p})\gamma\ \not{\ell}\gamma^{\nu}(\not{\ell}+\not{q})\gamma^{\rho}\gamma_5]$$

We get $r_{\rho}N^{\mu\nu\rho}$:

$$r_\rho N^{\mu\nu\rho} = \frac{1}{2} Tr[(\not\ell - \not p)\gamma \ \not\ell \gamma^\nu (\not\ell + \not q) \mathrm{r}_\rho \gamma^\rho \gamma_5] \qquad 4.15$$

Using the trace circle property we find:

$$r_\rho N^{\mu\nu\rho} = \frac{1}{2} Tr[\not\ell \gamma^\nu (\not\ell + \not q) \mathrm{r}_\rho \gamma^\rho (\not\ell - \not p)\gamma \ \gamma_5] \qquad 4.16$$

From the four momentum conservation p+q+r =0 we have r =-p-q so we write $r_\rho \gamma^\rho$ like :

$$\mathrm{r}_\rho \gamma^\rho = \not r = -(\not q + \not p) = -(\not\ell + \not q) + (\not\ell - \not p) \qquad 4.17$$

To find 4.16 we use

$$(\not\ell + \not q)\mathrm{r}_\rho \gamma^\rho (\not\ell - \not p) = (\not\ell + \not q)\left[-(\not\ell + \not q) + (\not\ell - \not p)\right](\not\ell - \not p) = (\ell + \mathrm{q})^2 (\not\ell - \not p) - (\ell - \mathrm{p})^2 (\not\ell + \not q) \qquad 4.18$$

Therefore we have[2]

$$r_\rho N^{\mu\nu\rho} = \frac{1}{2}(\ell + \mathrm{q})^2 \mathrm{Tr}[\not\ell \gamma^\nu (\not\ell - \not p)\gamma \ \gamma_5] - \frac{1}{2}(\ell - \mathrm{p})^2 \mathrm{Tr}[\not\ell \gamma^\nu (\not\ell + \not q)\gamma^\mu \gamma_5] = -2\mathrm{i}\varepsilon^{\alpha\nu\beta} [(\ell + \mathrm{q})^2 \ell_\alpha (\ell - \mathrm{p})_\beta - (\ell - \mathrm{p})^2 \ell_\alpha (\ell + \mathrm{q})_\beta]$$
$$= +2\mathrm{i}\varepsilon^{\alpha\nu\beta\mu}[(\ell + \mathrm{q})^2 \ell_\alpha p_\beta + (\ell - \mathrm{p})^2 \ell_\alpha q_\beta] \qquad 4.19$$

So the $r_\rho V^{\mu\nu\rho}$ becomes

$$r_\rho V^{\mu\nu\rho}(p,\ q,\ r) = -2g^3 \varepsilon^{\alpha\nu\beta} \int \frac{d^4\ell}{(2\pi)^4} \left[\frac{\ell_\alpha p_\beta}{(\ell - p)^2 \ell^2} + \frac{\ell_\alpha q_\beta}{(\ell + q)^2 \ell^2} \right] ? \ (p,\ ? \leftrightarrow \ ,\ \nu) + O(g^5)\ ? \qquad ?.20$$

If the integral is convergent we have the results which proportional to $\varepsilon^{\alpha\nu\beta\mu} p_\alpha p_\beta$ and to $\varepsilon^{\alpha\nu\beta\mu} q_\alpha q_\beta$ which equal zero[2] because of anti-symmetry tensor $\varepsilon^{\alpha\nu\beta\mu}$ therefore:

$$r_\rho V^{\mu\nu\rho}(p,\ q,\ r) = 0^? \qquad 4.21$$

Using the relation

$$iV^{\mu\nu\rho}(p,\ q,\ r) = -\frac{1}{2}(ig)^3 C^{\ \nu\rho}(p,\ q,\ r) + O(g5)$$

we have

$$r_\rho C^{\mu\nu\rho}(p,\ q,\ r) = 0^? \qquad 4.22$$

so from the relation

$$\langle \mathrm{p,q} | \partial_\rho \mathrm{j}^\rho_A(\mathrm{z}) | 0 \rangle = -\mathrm{ie}^2 \varepsilon_? \varepsilon'_\nu r_\rho \mathrm{C}^{\mu\nu\rho}(\mathrm{p,\ q,\ r})\mathrm{e}^{irz}\Big|_{r=-\mathrm{p}-\mathrm{q}}$$

we have

$$\langle \mathrm{p,q} | \partial_\rho \mathrm{j}^\rho_A(\mathrm{z}) | 0 \rangle = 0^? \qquad 4.23$$

Therefore the axial current $J^{\mu 5}$ is conserved $\partial_\mu J^{\mu 5} = 0$

But the integral 4.20 is not convergent, so we cannot decide if $\partial_\mu J^{\mu 5} = 0$.

The problem is the usual problem, which is, we have infinity degrees of freedom in the path integral, or infinity degrees of freedom associates with the quantum fluctuation .

If we recognize that the real world and so the real processes are convergent, so we replace the massless fermions propagator:

$$\bar{S}(\not{p}) = \frac{-\not{p}}{p^2} \qquad 4.24$$

With the modified propagator :

$$\bar{S}(\not{p}) = \frac{-\not{p}}{p^2}\frac{1}{1+\beta p^2} \qquad 4.25$$

We make $\beta = a^2 \to 0$ constant under the quantum fluctuation.

Using the propagator 4.25 instead of the propagator 4.24 in the integral 4.9, the integral 4.20 becomes convergent and equals zero, therefore $\partial_\mu J^{\mu 5}=0$. Which is hidden chiral symmetry.

That current can be written like $J^{\mu 5} = J^{\mu 5}_{free} + J^{\mu 5}_{pairing}$ so

$$\partial_\mu J^{\mu 5} = \partial_\mu J^{\mu 5}_{free} + \partial_\mu J^{\mu 5}_{pairing} = 0 \qquad 4.26$$

with $J^{\mu 5}_{free} = \bar{\psi}\gamma^\mu\gamma^5\psi$ which associates with SU(1) invariance for free fermions.

And $J^{\mu 5}_{pairing}$ relates to the fermions field dual behavior. For the quarks we relate that current to the flavor symmetry (chapter 9, Feynman diagrams), therefore we can generate these currents (4.26) to include different flavors q_i like $J^{\mu 5 \alpha}_{free} = \bar{Q}\gamma^\mu\gamma^5 T_2^\alpha Q$; $Q = \begin{pmatrix} q_i \\ q_j \end{pmatrix}$

Therefore we can think that the chiral symmetry is satisfied for both gauge invariance and flavor invariance together, so it is hidden symmetry, we can think that is related to the vacuum polarization and condensation under the quantum fluctuation.

5. Z_i parameters and Quarks Potential

We search for $-a^2p^2 \to 0$ for *timelike* and $a^2p^2 \to 0$ for *spacelike* but with making Z_i =constant so we can ignore the terms $\frac{a^2k^2}{1+a^2k^2}$ and $\frac{a^2p^2}{1+a^2p^2}$ in the propagators

$$\bar{\Delta}^{ab}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}\delta^{ab}}{k^2 - i\varepsilon}\left(1 - \frac{a^2k^2}{1+a^2k^2}\right) \; and \; \bar{S}_{ij}(\not{p}) = \frac{-\not{p}\delta_{ij}}{p^2 - i\varepsilon}\left(1 - \frac{a^2p^2}{1+a^2p^2}\right)$$

The indexes *a* and *b* gluons indexes, *i* and *j* color indexes and a is critical length.

To have the usual free quarks propagator, we see if we can make $-a^2p^2 \to 0$ with Z_i=constant, we begin with the quarks at high energy they become free particles, so it is good to assume Z_2=constant, and make $a^2p^2 << 1$

we have Z_2 eq1.18

$$Z_2 = 1 + C(R)\frac{\alpha_s}{2\pi}\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1-x_1}{[\beta f + (1-x_1-x_2)]} \; : \; \beta = a^2 \, and \; f = -x_1^2p^2 + x_1p^2 + x_1m_q^2 + x_2m_\gamma^2 \to -x_1^2p^2 + x_1p^2$$

For easy we ignore m_q and m_γ so

$$Z_2 = 1 + C(R)\frac{\alpha_s}{2\pi}\int_0^1 (1-x)\ln\left(1+\frac{1}{a^2p^2x}\right)dx = 1 + \frac{C(R)\alpha_s}{4\pi(a^2p^2)^2}\left[(a^2p^2)^2\ln\left(1+\frac{1}{a^2p^2}\right) - a^2p^2 + (2a^2p^2+1)\ln(a^2p^2+1)\right] : \; 5.1$$

To make Z_2 constant we assume for $-\mathrm{a}^2p^2<1$, and consider only the real part by using the property $\ln(x)= i\pi+\ln(x)$ we have

$$\frac{\alpha_s}{(-\mathrm{a}^2p^2)^2}\left[(-\mathrm{a}^2p^2)^2\ln\left(-1+\frac{1}{-\mathrm{a}^2p^2}\right)-\mathrm{a}^2p^2-(-2\mathrm{a}^2p^2-1)\ln\left(-(-\mathrm{a}^2p^2)+1\right)\right]=c \qquad 5.2$$

c: constant, we have the diagram

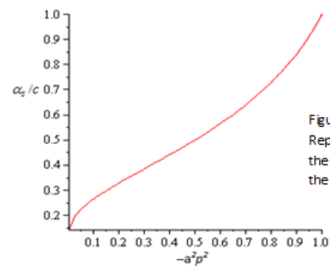


Figure 5.1
Representation the relation between the coupling constant α_s and $-\mathrm{a}^2p^2$ by the way of fixing Z_2

According to the figure 5.1 to make Z_2=constant we have the conditions:

The condition $-\mathrm{a}^2p^2<<1$ or $-\mathrm{a}^2p^2\to 0$ associates with $\alpha_s\to 0$ for the strong interaction, quarks with gluons, that is only at high energy, so to satisfy $-\mathrm{a}^2p^2<<1$, the length a must drop extremely by increasing the energy $-p^2$. By that we can think that the term $\mathrm{a}^2\mathrm{p}^2/(1+\mathrm{a}^2\mathrm{p}^2)$ is removed from high energy quarks and gluons propagators and have the usual free propagation.

Oppositely in non-strong interaction, like the electrodynamics interaction, the coupling constant α_e decreases by decreasing the energy $-p^2$, so the conditions Z_2=constant and $-\mathrm{a}^2p^2<<1$ are only at low energies ($p<<1/\mathrm{a}$), here we make the energy a^{-1} equal the energy scale M which appears in $\alpha_e(-p^2/M^2)$: $p^2/M^2<-1$ therefore always $-\mathrm{a}^2p^2<<1$ so we can ignore the terms $\mathrm{a}^2\mathrm{p}^2/(1+\mathrm{a}^2\mathrm{p}^2)$ and $\mathrm{a}^2\mathrm{k}^2/(1+\mathrm{a}^2\mathrm{k}^2)$ in non-strong interaction, like the electrodynamics interaction.

The problem gets stronger in low energy strong interaction where the coupling constant α_s *increases* with the energy $-p^2$ *decreasing*, but in this case, according to the diagram 5.1 (Z_2= constant) $-\mathrm{a}^2p^2$ must *increase* so it is possible to have $-\mathrm{a}^2p^2>1$.

Therefore the terms $\mathrm{a}^2\mathrm{p}^2/(1+\mathrm{a}^2\mathrm{p}^2)$ and $\mathrm{a}^2\mathrm{k}^2/(1+\mathrm{a}^2\mathrm{k}^2)$ take place in the low energy strong interaction, but when $-\mathrm{a}^2p^2>1$ we have $r<\mathrm{a}$ in the space. we can relate that to the quarks confinement: at low energy quarks there is a condition $r<\mathrm{a}$ with fixed length a we try to find the conditions for the length a, but we don't forget that is only at low energy quarks.

let us try to make the quarks masses independent on the energy $-p^2$, we have the relations $m_{0q}=Z_2^{-1}Z_m m_q$ and $\psi_0=\sqrt{Z_2}\psi$ if we make $Z_2^{-1}Z_m=\text{constant}$ so the mass m_q becomes independent on the energy $-p^2$. It is easier to assume $Z_2^{-1}Z_m=2$ so $m_{0q}=2m_q$=constant.

The Lagrange parameters Z_2 and Z_m for the quarks are like the electrons, the relations 1.13 and 1.14, for the quarks we have eq1.18 :

$$Z_2=1+C(R)\frac{g_s^2}{8\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{1-x_1}{[\beta f+(1-x_1-x_2)]} \ :\ \beta=\mathrm{a}^2 \qquad 5.3$$

$$Z_m=1+C(R)\frac{g_s^2}{8\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{2}{[\beta f+(1-x_1-x_2)]} \qquad 5.4$$

For the quarks R=3 and C(3)=4/3 , for $Z_m=2\,Z_2$ that becomes

$$1-\frac{4}{3}\frac{\alpha_s}{2\pi}\int_0^1 dx_1\int_0^{1-x_1} dx_2\frac{x_1}{[\beta f+(1-x_1-x_2)]}=0\ :\ \beta=\mathrm{a}^2 \text{ and } f=-x_1^2p^2+x_1p^2+x_1m_q^2+x_2m_\gamma^2$$

ignoring m_q and m_γ So $1-\frac{4}{3}\frac{\alpha_s}{2\pi}\int_0^1 dx_1\int_0^{1-x_1} dx_2\frac{x_1}{[\beta f+(1-x_1-x_2)]}=1+\frac{4}{3}\frac{\alpha_s}{2\pi}\int_0^1 x\ln\left(1+\frac{1}{\beta p^2x}\right)dx=0$

and $$\int_0^1 x\ln\left(1+\frac{1}{\beta p^2 x}\right)dx=\frac{1}{2(\beta p^2)^2}\left[-ln\left(\frac{\beta p^2+1}{\beta p^2}\right)+\beta p^2+ln\left(\frac{1}{\beta p^2}\right)+(\beta p^2)^2ln\left(\frac{\beta p^2+1}{\beta p^2}\right)\right]$$

Therefore

$$1-\frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1}{2(\beta p^2)^2}\left[-ln\left(\frac{\beta p^2+1}{\beta p^2}\right)+\beta p^2+ln\left(\frac{1}{\beta p^2}\right)+(\beta p^2)^2ln\left(\frac{\beta p^2+1}{\beta p^2}\right)\right]=0 \qquad 5.5$$

for $\beta p^2=a^2p^2<<1$ spacelike, we approximate 5.5 to $1-\frac{4}{3}\frac{\alpha_s}{2\pi}\ln(1/\mathrm{a}^2p^2)\approx 0$

as we found for Z_2= constant that condition $\mathrm{a}^2p^2<<1$ is satisfied at high energy $\alpha_s\to 0$.

For $\beta p^2=\mathrm{a}^2p^2>1$ spacelike, we approximate 5.5 to $1-\frac{4}{3}\frac{\alpha_s}{2\pi}\frac{1}{\mathrm{a}^2p^2}\approx 0$

that condition is satisfied at low limited energy, α_s is higher so a^2p^2 increases and reaches $\mathrm{a}^2p^2>1$ when the quarks energy drops as we found for Z_2= constant.

Therefore the terms $\mathrm{a}^2\mathrm{p}^2/(1+\mathrm{a}^2\mathrm{p}^2)$ and $\mathrm{a}^2\mathrm{k}^2/(1+\mathrm{a}^2\mathrm{k}^2)$ take place only in the low energy strong interaction with fixing Z_2 and m.

With that there is critical point in low limited energy strong interaction, it is $-\mathrm{a}^2p^2=1$, when $\mathrm{a}^2p^2<1$ the quarks are free particles, high energy.

And when $\mathrm{a}^2p^2>1$ the quarks become confinement $p^2>1/\mathrm{a}^2$: $r<\mathrm{a}$ low limited energy quarks.

For $\mathrm{a}^2\mathrm{p}^2\to 0$ eq 5.2 becomes, in *spacelike*, $\alpha_s\ln\left(p^2\mathrm{a}^2\right)=\text{constant}=-c<0$

So $$\mathrm{a}^2=\frac{1}{p^2}e^{\frac{-c}{\alpha_s}}\to 0\ :\ p^2>>\Lambda_{QCD}^2\ \textit{and}\ \alpha_s\to 0\ \textit{decoupling} \qquad 5.6$$

So we can ignore the terms $\frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}$ and $\frac{\mathrm{a}^2p^2}{1+\mathrm{a}^2p^2}$ in the propagators

$$\overline{\Delta}_{\mu\nu}^{ab}(k^2)=\frac{g_{\mu\nu}\delta^{ab}}{k^2-i\varepsilon}\left(1-\frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}\right)\ \textit{and}\ \overline{S}_{ij}(\not p)=\frac{-\not p\delta_{ij}}{p^2-i\varepsilon}\left(1-\frac{\mathrm{a}^2p^2}{1+\mathrm{a}^2p^2}\right)$$

But when the energy drops a^2 and α_s increase, we make α_{0S} and a_0 the highest values and $\frac{2\alpha_{0s}}{3\mathrm{a}_0^2}=\sigma$ the string tension, we find it in the low energy potential 5.13 , in general we fix $\frac{2\alpha_s}{3\mathrm{a}^2}=\sigma$ as a string tension.

With that we must give the modifying terms $\frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}$ and $\frac{\mathrm{a}^2p^2}{1+\mathrm{a}^2p^2}$ a physical meaning, we find the Lagrange terms which associate with them, that is in chapter 9, we find there is a field dual behavior, free field behavior makes possibility for separating the particles and the composition behavior makes possibility for condensation them.

The area a^2 is a Lattice area in space-time, $\mathrm{a}^2=\mathrm{a}_\tau\mathrm{a}_r$ and $\mathrm{a}_\tau=\mathrm{a}_r=\mathrm{a}$

we have from the string tension σ: $\sigma = \mathrm{a}_1 = \frac{2\alpha_s}{3\mathrm{a}^2}$ (eq 5.13)

$$\sigma = \frac{g_s^2}{6\pi\mathrm{a}^2} \;\rightarrow \frac{g_s}{\mathrm{a}} = \sqrt{6\pi\sigma} = \text{constant} \qquad 5.7$$

With that the behavior of the length a like the behavior of the coupling constant g_s for the Quarks and Gluons strong interaction, the coupling constant $g_s \rightarrow 0$ at high energies, gluons quarks decoupling so a$\rightarrow$0

5.1 The quarks static potential at low limited energy

For the strong interaction we modify the quarks and gluons propagators like:

$$\overline{\Delta}_{\mu\nu}^{ab}(k^2) = \frac{g_{\mu\nu}\delta^{ab}}{k^2 - i\varepsilon}\left(1 - \frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}\right) \;:\; k^2 = \vec{k}^2 - k_0^2 \;\; and \;\; \overline{S}_{ij}(\not{p}) = \frac{-\not{p}\delta_{ij}}{p^2 - i\varepsilon}\left(1 - \frac{\mathrm{a}^2p^2}{1+\mathrm{a}^2p^2}\right) \;:\; p^2 = \vec{p}^2 - p_0^2 \qquad 5.8$$

The indexes *a* and *b* gluons indexes, *i* and *j* color indexes and a is critical length.

In the beginning we consider the length a as constant parameter, its unit energy^{-1} we use the energy unit, $[r]$=energy^{-1}, c=$\hbar$=1.

And to make the Lagrange parameters Z_i and so the quarks masses constants, we need to make $\beta=\mathrm{a}^2=\mathrm{a}_\tau\,\mathrm{a}_r$ depend on the energy $-p^2$ and have $-\mathrm{a}^2p^2<<1$, that was right for high energy, free quarks and $r>\mathrm{a}\rightarrow 0$ which is not confinement, it is the quarks and gluons decoupling.

The problem appears at low energy when $-\mathrm{a}^2p^2>1$ so $r<\mathrm{a}$ which is the confinement, we find the potential and see the case $r<\mathrm{a}$. We make *r* the distance between the two interacted quarks.

We define that potential in momentum space $\tilde{V}(k)$ using *M* matric element for quark-quark(gluons exchanging) interaction, with $\omega=k_0=0$(like Born approximation to scattering amplitude in non-relativistic quantum mechanics[1])

$$iM = -i\tilde{V}(k)J^\mu(p_2', p_2)J_\mu(p_1', p_1) \qquad 5.9$$

with the transferred current $J^\mu(p', p) = \overline{u}(p')\gamma^\mu u(p)$ with spinor states *u(p)* include the helicity states.

We find *M* matrix element using the Feynman diagrams for quark-quark gluons exchanging using color representation for one quark like

$$u(p)_{color\otimes spinor} = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}u(p)_{spinor}$$

For distinguishable quarks we have

$$iM = \overline{u}^i(p_2')ig_s\gamma^\mu(T^a)_i{}^j u_j(p_2)\frac{\Delta_{\mu\nu}^{ab}(k^2)}{i}\overline{u}^k(p_1')ig_s\gamma^\nu(T^b)_k{}^\ell u_\ell(p_1) \qquad 5.10$$

Using Gell-Mann matrices, the matrices $T^a=\lambda^a$: $\lambda_1,\ldots,\lambda_8$ consider them as SU(3) generators, and $k=p_2'-p_2=p_1-p_1'$ using the modified gluons propagator 5.8 we have

$$iM = \sum_{ijk\ell} ig_s^2\overline{u}^i(p_2')\gamma^\mu(T^a)_i{}^j u_j(p_2)\frac{g_{\mu\nu}\delta^{ab}}{k^2}\left(1 - \frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}\right)\overline{u}^k(p_1')\gamma^\nu(T^b)_k{}^\ell u_\ell(p_1) \qquad 5.11$$

to sum over the color indexes *i*, *j* with the color representation like above and over gluon index a we write

$$\sum_{ij}\bar{u}^i(p_2')\gamma^\mu (T^a)_i^{\ j} u_j(p_2) = \bar{u}(p_2')\gamma^\mu \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\end{pmatrix}(T^a)\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}u(p_2)$$

And
$$\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\end{pmatrix}(T^a)\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix} = \frac{1}{3}\sum_{ij}(T^a)_i^{\ j}$$

Therefore the M matrix element becomes

$$M = \frac{1}{9}\sum_a\left(\sum_{ij}(T^a)_i^{\ j}\right)^2 g_s^2 \bar{u}(p_2')\gamma^\mu u(p_2)\frac{1}{k^2}\left(1-\frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}\right)\bar{u}(p_1')\gamma_\mu u(p_1)$$

The Gell-Mann matrices with nonzero sum of the elements are

$$\lambda_1 = \begin{pmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 0\end{pmatrix},\ \lambda_4 = \begin{pmatrix}0 & 0 & 1\\0 & 0 & 0\\1 & 0 & 0\end{pmatrix}\ and\ \lambda_6 = \begin{pmatrix}0 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{pmatrix}$$

So $\sum_a\left(\sum_{ij}(T^a)_i^{\ j}\right)^2 = 3\cdot(2)^2 = 12$ Therefore we have

$$M = \frac{12g_s^2}{9}\frac{1}{k^2}\left(1-\frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}\right)\bar{u}(p_2')\gamma^\mu u(p_2)\bar{u}(p_1')\gamma_\mu u(p_1) \qquad 5.12$$

So we have the potential $\tilde{V}(k)$ in momentum space as we defined

$$iM = -i\tilde{V}(k)J^\mu(p_2', p_2)J_\mu(p_1', p_1) = i\frac{12g_s^2}{9}g_s^2\bar{u}(p_2')\gamma^\mu u(p_2)\frac{1}{k^2}\left(1-\frac{k^2}{k^2+1/\mathrm{a}^2}\right)\bar{u}(p_1')\gamma_\mu u(p_1)$$

With the transferred currents $J^\mu(p_2', p_2) = \bar{u}(p_2')\gamma^\mu u(p_2)$ and $J^\mu(p_1', p_1) = \bar{u}(p_1')\gamma^\mu u(p_1)$

So we have
$$\tilde{V}(k) = -\frac{4g_s^2}{3}\frac{1}{k^2}\left(1-\frac{k^2}{k^2+1/\mathrm{a}^2}\right)$$

Making Fourier transformation to the space XYZ, we have the potential $U(x)$ with k_0=0 like electric potential[1].

$$U(x) = \int\frac{d^3k}{(2\pi)^3}\tilde{V}(k)\, e^{ik\cdot x} = -\frac{4g_s^2}{3}\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2}\left(1-\frac{k^2}{k^2+1/\mathrm{a}^2}\right)e^{ik\cdot x} = -\frac{4g_s^2}{3\cdot 4\pi r}\left(1-\exp(-\frac{r}{\mathrm{a}})\right);\quad r=\sqrt{x^2+y^2+z^2}$$

For r<a :

$$U(r) = -\frac{4g_s^2}{3\cdot 4\pi r}\left[1-\exp(-\frac{r}{\mathrm{a}})\right] = u_0 + \mathrm{a}_1 r + \mathrm{a}_2 r^2 + \ldots.. \qquad 5.13$$

with $u_0 = -\frac{4}{3}\frac{g_s^2}{4\pi\mathrm{a}} = -\frac{4\alpha_s}{3\mathrm{a}},\ \mathrm{a}_1 = \sigma = \frac{g_s^2}{3\cdot 2\pi\mathrm{a}^2} = \frac{2\alpha_s}{3\mathrm{a}^2},\ \mathrm{a}_2 = \frac{4\alpha_s}{3\cdot 6\mathrm{a}^3}$: $\alpha_s = g_s^2/4\pi$

To fix $u_0 = -4\alpha_s/3\mathrm{a}$ we write it like $u_0 = -\frac{4\alpha_s}{3\mathrm{a}} = -\frac{4\alpha_s}{3\mathrm{a}^2}\mathrm{a} = -2\sigma\mathrm{a}$ with fixing the length a at low energy.

That potential appears at low limited energy and prevents the quarks from spreading away, r<a so it holds the quarks inside the Hadrons. But the starting from the high energies, although the quarks masses are small but they are created only at high energies where they are free and by dropping the energy the situation r<a

appears, the constant β=a^2 would run and becomes higher at low energies, so have $-a^2k^2>1$: $r<a$ which is the confinement.

We use that potential to study the quarks plasma, condensation,…

The confinement(*low energy*) means when $r\to a$ the two interacted quarks kinetic energy becomes zero (ignore the quark mass), therefore the highest kinetic energy can the quark get equals σa which relates to the potential $U(r)= u_0+\sigma r+...$ for $r<a$(at low limited energy).

We can make $U(r)$ the potential for all quarks in $r<a$, $\sigma\to\Sigma\sigma$ and consider r as average distance between the interacted quarks, so the energy σa becomes the highest kinetic energy of all quarks(consider them as free particles in volume b^3), therefore the quarks (massless) energy

$$\varepsilon=\sum_i \varepsilon_i=\sum_i P_i=2\pi\sum\sqrt{\left(\frac{n_1}{b_1}\right)^2+\left(\frac{n_1}{b_2}\right)^2+\left(\frac{n_1}{b_2}\right)^2}=\sigma a=\frac{N}{a}:\quad c=\hbar=1 \qquad 5.14$$

With N=1, 2,…. is frozen quarks energy quantization number . With that we have $a^2=\frac{N}{\sigma}$ therefore the area a^2 is quantized: $\Delta a^2=1/\sigma$ this area associates with energy quanta, we try to find it. using $a=\sqrt{\frac{N}{\sigma}}$ we have: $\varepsilon=\frac{N}{a}\to\varepsilon^2=N\sigma$ so the square total energy ε^2 is quantized, using $N=\sigma a^2$ we have

$$\varepsilon^2=\sigma a^2\sigma=\sigma^2a^2 \ \ therefore \quad \frac{\varepsilon^2}{a^2}=\sigma^2\ \to\ \frac{\varepsilon}{a}=\sigma \qquad 5.15$$

So, in the strong interaction the low energy is carried on the space length a .

6. Quarks energy renormalization

We find, the quarks are totally free at high energy but at low limited energy there is a potential $U(r)$ takes place with the condition $r<a$

$$U(r)=u_0+\sigma r+\ldots..\ :\ r<a\ and\quad u_0=-\frac{4\alpha_s}{3a}=-2a\sigma<0 \qquad 6.1$$

but when $r\to a$: $U\to -\sigma a<0$ so the total energy becomes negative because of the energy u_0 therefore we must remove it and have normed states, to do that we make a shift in the distance r between two neighbor quarks like: $r\to\delta r+2a$ setting this in the potential we have:

$$U(r)\to\ u_0+\sigma(\delta r+2a)+\ldots=-2\sigma a+\sigma(\delta r+2a)+\ldots=-2\sigma a+2\sigma a+\sigma\delta r+\ldots=\sigma\delta r+\ldots..>0 \qquad 6.2$$

With that the length a has smallest non-zero value $a_0=2a$ so we can't have $r\to0$ (at low limited energy)

If we assumed that the $U(0)=0$ is the ground states potential of the quarks with the fixed distance between them $a_0=2a$ therefore we expect that the composited quarks energies proportional to σa_0(frozen quarks inside the hadrons). So the quarks loss the energy σa_0 .

To do that we begin with the initial states where $r<a$ with non fixed a and make it $r<2a=a_0$ fixing the length a between them.

The quarks are massless, so for each quark q_i the energy $E_{qi}=P_{qi}$: c=1, in volume b^3 the momentum $p_{x,q_i}=\frac{2\pi n_i}{b_x}$; $n_i=1,2,....$ so the energy

$$\varepsilon=\sum_i \varepsilon_i=\sum_i P_i=2\pi\sum\sqrt{\left(\frac{n_1}{b_1}\right)^2+\left(\frac{n_1}{b_2}\right)^2+\left(\frac{n_1}{b_2}\right)^2}=\sigma \mathrm{a}=\frac{N}{\mathrm{a}} \qquad 牋 \qquad 6.3$$

With $b^2=(b_1)^2+(b_2)^2+(b_3)^2$ and N=1, 2,…. is the lattice number and energy quantization number.

The quarks energy $\varepsilon=\frac{N}{\mathrm{a}}=\sigma\mathrm{a}$ is defined in the situation $0<r<\mathrm{a}$ so we translate to the stationary situation $r\longrightarrow\delta r+\mathrm{a}_0$ so we must fix a like $\mathrm{a}_0+\delta\mathrm{a}$ and $N\longrightarrow N_0+\delta N$ with this renormalization the quarks square energy:

$$\varepsilon^2=N\sigma=\frac{N}{\mathrm{a}}\sigma\mathrm{a}\rightarrow\frac{N_0+\delta N}{\mathrm{a}_0}\sigma\mathrm{a}_0\rightarrow\frac{N_0}{\mathrm{a}_0}\sigma\mathrm{a}_0+\frac{\delta N_0}{\mathrm{a}_0}\sigma\mathrm{a}_0 \qquad 6.4$$

The term $\frac{N_0}{\mathrm{a}_0}\sigma\mathrm{a}_0$ is constant and can be written like $(\sigma\mathrm{a}_0)^2+...$ where we imposed $\frac{N_0}{\mathrm{a}_0}=\sigma\mathrm{a}_0+\delta$ so the square energy becomes $\varepsilon^2=\frac{\delta N_0}{\mathrm{a}_0}\sigma\cdot\mathrm{a}_0+(\sigma\cdot\mathrm{a}_0)^2+\cdots$

We can write $\varepsilon^2=(\delta p)^2+m^2_{condensed}$ where the square total quarks momentum $(\delta p)^2=\frac{\delta N_0}{\mathrm{a}_0}\sigma\mathrm{a}_0+\cdots$ and the condensed quarks mass $m_{condensed}=\sigma\mathrm{a}_0$

We make the energy $\sigma\mathrm{a}_0$ equals the hadron mass, like the proton $\sigma\mathrm{a}_0$=938*Mev* three quarks.

There is always oscillation $\delta\mathrm{a}\neq 0$(quantum fluctuation around a_0) inside the volume(b^3), with that, the quarks of different hadrons can interact and have the nuclear attractive potential $\delta u_0=-\sigma\delta\mathrm{a}<0$ but the high pressure controls that, so the potential strength is the same for all nucleuses and independent on the number of the nucleons.

As we will see in Feynman diagrams there are always two condensed quarks (pairing appears as scalar particle with mass $1/\mathrm{a}_0$) associate with free quarks, that as we imposed due to the negative energy $\delta u_0<0$.

We can determine σ by determining a_0 . Because of the dual quarks behavior, we find the value $1/\mathrm{a}_0$ equals the pion mass≈0.135*Gev* but in our calculations(*Quarks Condensation phase, hadrons*) it is suitable to make $1/\mathrm{a}_0$=120*Mev*. Here we expect $\sigma\backsim(\sigma\mathrm{a}_0)^2$ so if we set $\sigma=\frac{5}{12\pi}(\sigma\mathrm{a}_0)^2$ we have

$$\frac{1}{\mathrm{a}_0}=\frac{\sigma}{\sigma\mathrm{a}_0}=\frac{1}{\sigma\mathrm{a}_0}\frac{5}{12\pi}(\sigma\mathrm{a}_0)^2=\frac{5}{12\pi}\sigma\mathrm{a}_0=\frac{5\cdot 0.938}{12\pi}=0.124\,Gev$$

But for the quarks we make (the string tension) like $\sigma=\frac{g_q}{4\pi}\mu_0^2$

Where μ_0 is free quarks chemical potential and g_q is quarks degeneracy number. We fix $1/\mathrm{a}_0$=120*Mev* with g_q=12, the right values are 135–140 *Mev* the pions masses, but in our calculations(*Quarks Condensation phase, hadrons*) it is more suitable to use 120*Mev*(T_c = 111.4*mev* with g_q=12), we can make 135–140 *Mev* but we have to change g_q(*Confinement phase*) so changing T_c to have the same results.

7. Nuclear potential

we assume that the potential between the nucleons relates to the quarks potential but without confinement and the quarks condensation processes is ended when $E_H+U(r)>0$; $E_H>0$ hadrons energy and $U(r)<0$ quarks potential, then the hadrons interact by the same $U(r)$ (quarks potential) but that potential $U(r)$ becomes usual potential(without confinement).

$$\mathrm{U}(r) = -\frac{\alpha_s}{r}\left[1-\exp(-\frac{r}{\mathrm{a}})\right] \qquad 7.1$$

α_s and a become constants(no running).

Because of the dual behavior of the quarks filed which means for any two quarks interaction, the quarks composite and give scalar charged particles like the Pions π^-, π^0, π^+ and because of their quantized charges −1, 0, +1 we expect the hadrons charges also quantized –Q,–Q+1, …, 0, +1,…, +Q that quantization relates to the field dual behavior of the quarks in different hadrons.

we assumed that the interaction between the nucleons relates to the quarks interaction, so the nucleons potential inside the nucleus can be written like:

$$\mathrm{U}(r)_{nuc} = -\frac{\alpha}{r}\left[1-\exp(-\frac{r}{\mathrm{a}})\right] \qquad 7.2$$

the value of $U(r)$ in $r=0$: $\mathrm{U}(0)_{nuc} = -\frac{\alpha}{\mathrm{a}}$ 7.3

We need conditions in the case $r>$a to determine $U(0)_{nuclear}$. Below we assume the relation:

$$\mathrm{a} = \frac{1}{m^*\alpha} \to \frac{\alpha}{\mathrm{a}} = m^*\alpha^2 \qquad 7.4$$

Also we have $m^*\alpha^2$=0.088amu, so $\mathrm{U}(0)_{nuc} = -\frac{\alpha}{\mathrm{a}} = m^*\alpha^2 = -0.088amu = -82Mev$ 7.5

The energy -82*Mev* is smallest nuclear potential.

If we use the potential 7.2 in Schrödinger equation for one nucleon we have:

$$-\frac{\nabla^2}{2m^*}\psi + U(r)\psi = -\frac{\nabla^2}{2m^*}\psi - \frac{\alpha}{r}\left(1-\exp(-\frac{r}{\mathrm{a}})\right)\psi = E\psi \;\; with\ \hbar = c = 1\ and\ m^*\text{: nucleon effective mass} \qquad 7.6$$

We solve that equation approximately using the variational method as known in the quantum mechanics and make a=1/m*α For arbitrary α^* we solve the equation

$$-\frac{\nabla^2}{2m^*}\psi - \frac{\alpha^*}{r}\psi = E\psi \qquad 7.7$$

Its solution like the hydrogen atom solution

$$\psi(r,\theta,\varphi,\alpha^*)_{n,\ell,m} = R(r,\alpha^*)_{n,\ell}Y_{\ell,m}(\theta,\varphi)\ with\ \varepsilon_n(\alpha^*) = -\frac{1}{2}\frac{m^*\alpha^{*2}}{n^2}$$

We use that in the eq7.6 and minimum the energy E

$$E = \langle\psi(\alpha^*)| -\frac{\nabla^2}{2m^*} - \frac{\alpha}{r} + \frac{\alpha}{r}e^{-r/\mathrm{a}}|\psi(\alpha^*)\rangle = \langle\psi(\alpha^*)| -\frac{\nabla^2}{2m^*} - \frac{\alpha^*}{r} + \frac{\alpha^*-\alpha}{r} + \frac{\alpha}{r}e^{-r/\mathrm{a}}|\psi(\alpha^*)\rangle$$

That becomes

$$E_n(\alpha^*) = \varepsilon_n(\alpha^*) + \langle\psi(\alpha^*)|\frac{\alpha^*-\alpha}{r} + \frac{\alpha}{r}e^{-r/\mathrm{a}}|\psi(\alpha^*)\rangle \qquad 7.8$$

Where

$$\langle\psi(\alpha^*)|\frac{\alpha^*-\alpha}{r} + \frac{\alpha}{r}e^{-r/\mathrm{a}}|\psi(\alpha^*)\rangle = \int d^3r\left(R_{n,\ell}(r,\alpha^*)Y_{\ell,m}(\theta,\varphi)\right)^*\left(\frac{\alpha^*-\alpha}{r} + \frac{\alpha}{r}e^{-r/\mathrm{a}}\right)R_{n,\ell}(r,\alpha^*)Y_{\ell,m}(\theta,\varphi)$$

That becomes

$$\int_0^\infty dr\left(R_{n,\ell}(r,\alpha^*)\right)^*\left(\frac{\alpha^*-\alpha}{r}+\frac{\alpha}{r}e^{-r/a}\right)R_{n,\ell}(r,\alpha^*)$$

R(r) is real so we have

$$\langle\psi(\alpha^*)|\frac{\alpha^*-\alpha}{r}+\frac{\alpha}{r}e^{-r/a}|\psi(\alpha^*)\rangle=\int_0^\infty\left(R_{n,\ell}(r,\alpha^*)\right)^2\left(\frac{\alpha^*-\alpha}{r}+\frac{\alpha}{r}e^{-r/a}\right)dr$$

The energy becomes

$$E_{n,\ell}(\alpha^*)=\varepsilon_n(\alpha^*)+\int_0^\infty\left(R_{n,\ell}(r,\alpha^*)\right)^2\left(\frac{\alpha^*-\alpha}{r}+\frac{\alpha}{r}e^{-r/a}\right)dr$$

We calculate that for n=1, 2, 3 and minimum the energy: $\frac{\partial}{\alpha^*}E_{n,\ell}(\alpha^*)=0$ so omitting α^* . We use the normalized radial wave functions for n=1, 2, 3

$$R_{1,0}(r,\alpha^*)=2\left(a^*\right)^{-3/2}e^{-r/a^*}\ \textit{with}\ a^*=1/m^*\alpha^*$$

$$R_{2,0}(r,\alpha^*)=\frac{1}{\sqrt{2}}\left(a^*\right)^{-3/2}\left(1-\frac{r}{2a^*}\right)e^{-r/2a^*}$$

$$R_{2,1}(r,\alpha^*)=\frac{1}{\sqrt{24}}\left(a^*\right)^{-3/2}\frac{r}{a^*}e^{-r/2a^*}$$

$$R_{3,0}(r,\alpha^*)=\frac{2}{\sqrt{27}}\left(a^*\right)^{-3/2}\left(1-\frac{2r}{3a^*}+\frac{2r^2}{27\left(a^*\right)^2}\right)e^{-r/3a^*}$$

We have

$$E_{n,\ell}=m^*\alpha^2 b_{n,\ell}$$

$$1s:E_{1,0}=m^*\alpha^2(-0.25)=-0.25m^*\alpha^2$$

$$2s:E_{2,0}=m^*\alpha^2(-0.088)=-0.088m^*\alpha^2$$

$$2p:E_{2,1}=m^*\alpha^2(-0.109)=-0.109m^*\alpha^2$$

$$3s:E_{3,0}=m^*\alpha^2(-0.044)=-0.044m^*\alpha^2$$

We fill them like: n=1, ℓ=0: $1S^2 1S^2$ maximum 4 nucleons, two protons and two neutrons.

n=2, ℓ=0,1: $2S^2 2S^2 2P^6 2P^6$ maximum 16 nucleons, eight protons and eight neutrons.

n=3, ℓ=0: $3S^2 3S^2$ 4 nucleons, two protons and two neutrons.

Therefore the associated binding energy for the nucleus $E_{binding}$ becomes

$$E_b=\sum_{n,\ell}(Z_{n,\ell}+N_{n,\ell})E_{n,\ell}=m^*\alpha^2\sum_{n,\ell}(Z_{n,\ell}+N_{n,\ell})\,b_{n,\ell}\prec 0 \qquad 7.9$$

To calculate $m^*\alpha^2$, we calculate the right binding energy $\Delta m=Zm_p+Nm_N-m$ where m is the measured nucleus mass and m_p free proton mass, m_N free neutron mass.

We fit $E_{binding}$ equation 7.9 with $\Delta m=Zm_p+Nm_N-m$, but we add constant $\Delta>0$ to it

$$E_b=m^*\alpha^2\sum_{n,\ell}(Z_{n,\ell}+N_{n,\ell})\,b_{n,\ell}+\Delta$$

For that, we have the figure 7.1

From that figure we have the fitting

$$\Delta m(amu) = 0.088\sum_{n,\ell}(Z_{n,\ell} + N_{n,\ell})(-b_{n,\ell}) - 0.067$$

Or

$$-\Delta m(amu) = 0.088\sum_{n,\ell}(Z_{n,\ell} + N_{n,\ell})\, b_{n,\ell} + 0.067 \prec 0$$

Comparing with

$$E_b = m^*\alpha^2\sum_{n,\ell}(Z_{n,\ell} + N_{n,\ell})\, b_{n,\ell} + \Delta$$

We have $m^*\alpha^2$=0.088*amu* and Δ=0.068*amu* .

For m^*=1*amu* the constant α=0.3, the range a=1/m*α≈7·10^{-16}*m* .

He	1	0.0292
Li	1.176	0.0348
Be	1.463	0.0603
B	1.573	0.067
C	1.793	0.096
N	2.012	0.098
O	2.232	0.1270
F	2.562	0.1508
Ne	2.672	0.1674
Na	2.804	0.1932

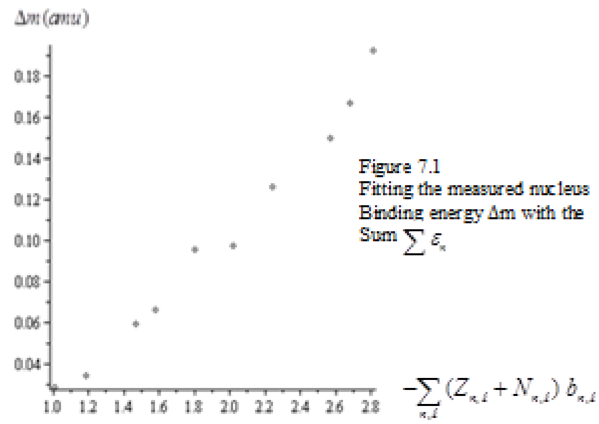


Figure 7.1
Fitting the measured nucleus Binding energy Δm with the Sum $\sum \varepsilon_\kappa$

We try to solve the eq7.6 using the harmonic oscillation solutions.
In the spherical coordinates we have

$$-\frac{1}{2m^*}\frac{d}{r^2dr}(r^2\frac{d}{dr})R(r) + \left[U(r) + \frac{1}{2m^*}\frac{\ell(\ell+1)}{r^2}\right]R(r) = ER(r) \qquad 7.10$$
$$with \;\; \psi_{n,\ell,m}(r,\theta,\varphi) = R_{n,\ell}(r)Y_{\ell,m}(\theta,\varphi)$$

making *R(r)=u(r)/r* we have:

$$-\frac{1}{2m^*}\frac{d^2}{dr^2}u(r) + \left[U(r) + \frac{1}{2m^*}\frac{\ell(\ell+1)}{r^2}\right]u(r) = Eu(r)$$

we have

$$\hat{H} = -\frac{1}{2m^*}\frac{d^2}{dr^2} + U(r) + \frac{1}{2m^*}\frac{\ell(\ell+1)}{r^2} = -\frac{1}{2m^*}\frac{d^2}{dr^2} - \frac{\alpha}{r}\left(1 - \exp(-\frac{r}{\mathrm{a}})\right) + \frac{1}{2m^*}\frac{\ell(\ell+1)}{r^2} \qquad 7.11$$

We can write:

$$\hat{H} = -\frac{1}{2m^*}\frac{d^2}{dr^2} + U(r)_{eff}$$

with the effective potential($\ell \neq 0$)and 2αm^*a>>1:

$$U(r)_{eff}=-\frac{\alpha}{r}\left(1-\exp(-\frac{r}{\mathrm{a}})\right)+\frac{1}{2m^{*}}\frac{\ell(\ell+1)}{r^{2}} \qquad 7.12$$

For $2\alpha m^*\mathrm{a}>>1$ as expected, that potential has behavior like the diagram:

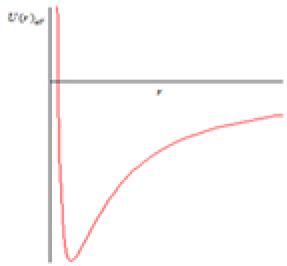


Diagram 7.2
Representation the potential

So for $r/\mathrm{a}<1$ as expected we have harmonic oscillation(is like it)in spherical coordinates:

$$U(\delta r)_{eff}\approx -u_{0}+\frac{1}{2}m^{*}\omega^{2}(\delta r)^{2}+.... \ : \ \ell\neq 0 \qquad 7.13$$

Making $r=\mathrm{a}x$ in the potential 7.12 we have

$$U(x)_{eff}=-\frac{\alpha}{\mathrm{a}x}\left(1-\mathrm{e}^{-x}\right)+\frac{\ell(\ell+1)}{2m^{*}\mathrm{a}^{2}}\frac{1}{x^{2}}$$

We had $\alpha/\mathrm{a}=m^*\alpha^2=0.088amu$ and $m^*\mathrm{a}^2=m^*(m^*\alpha)^{-2}=(m^*\alpha^2)^{-1}$ so

$$U(x)_{eff}=m^{*}\alpha^{2}\left(-\frac{1}{x}\left(1-\mathrm{e}^{-x}\right)+\frac{\ell(\ell+1)}{2}\frac{1}{x^{2}}\right) \qquad 7.14$$

Comparing with 7.13 for the orbital (P): $\ell=1$ we have $u_0=0.2m^*\alpha^2=0.0176amu=16Mev$ and $\omega=0.252m^*\alpha^2=20.5Mev$.
For $\ell=2$ we have $u_0=0.08m^*\alpha^2=6.55Mev$ and $\omega=0.068m^*\alpha^2=5.62Mev$.

If there is balance situation in $r=\mathrm{a}$ so that potential can be modified near $r=\mathrm{a}$ like:

$$U(r)=-\frac{\alpha}{2\mathrm{a}}+\frac{\alpha}{4\mathrm{a}^{3}}(r-\mathrm{a})^{2}+..... \ \approx -\frac{\alpha}{2\mathrm{a}}+\frac{1}{2}m^{*}\omega^{2}(r-\mathrm{a})^{2} \ : \ \omega^{2}=\frac{\alpha}{2m^{*}\mathrm{a}^{3}} \qquad 7.15$$

So the Schrödinger equation for that potential is harmonic oscillation in spherical coordinates, the solution (Abramowitz, Stegun 1964):

$$u_{k\ell}(r)=N_{k,\ell}r^{\ell+1}e^{-\nu r^{2}}L_{k}^{\ell+1/2}(2\nu r^{2}): \ \nu=m\omega/2 \qquad 7.16$$
$$with \ \varepsilon_{k\ell}=-V_{0}+\omega(2k+\ell+3/2): \quad V_{0}=\frac{\alpha}{2\mathrm{a}} \ and \ \omega=\sqrt{\frac{\alpha}{2m^{*}\mathrm{a}^{3}}}$$

That perturbed interaction energy for one nucleon in the nucleus becomes:

$$\varepsilon_{k\ell} = -\frac{m^*\alpha^2}{2} + \omega(2k+\ell+3/2):\ \omega = \sqrt{\frac{\alpha^3}{2m^*\alpha^2 a^3}} = \sqrt{\frac{1}{2m^*\alpha^2}\left(\frac{\alpha}{a}\right)^3} = \sqrt{\frac{1}{2m^*\alpha^2}\left(m^*\alpha^2\right)^3} = \frac{m^*\alpha^2}{\sqrt{2}} \qquad 7.17$$

For $m^*\alpha^2$=0.088*amu* we have ω=58*Mev*.

8. Quark Magnetic and angular momentum inside the hadrons

Because of the quarks energy renormalization we saw that the ground state distance a_0 between the quarks is fixed(at low limited energy), it equivalents to zero quarks energy (frozen quarks inside the hadrons).

If we use the classical definition of the Magnetic moments μ:

$$\mu = \frac{1}{2}\sum_{q_i} e_{q_i} r_i \times v_i \qquad 8.1$$

e_{qi} quark charge, r_i and v_i position and velocity.

We considered the quarks massless, so the velocity equals the light velocity c=$\hbar$=1 so:

$$\mu = \frac{1}{2}\sum_{q_i} e_{q_i} < r_c > \qquad 8.2$$

Where $<r_c>$ is the average distance of the quark from the rotation center.

In the proton there are three quarks with condensation energy σa_0=0.938 *Gev* so the energy $\sigma a_0/3$ is for each quark, therefore we expect the distance $a_0/3$ is the average distance for the quark in the baryons, so

$$< r_c > = \frac{a_0}{3} = \frac{1}{\mu^*} \qquad 8.3$$

Where the quark appears with high mass μ*, that because of the potential. So the quark magnetic moments inside the proton or the neutron becomes $\mu_{q_i} = \frac{1}{2}\frac{e_{q_i}}{\mu^*} = \frac{e_{q_i}}{e}\frac{e}{2\mu^*} = \frac{e_{q_i}}{e}\frac{m_p}{\mu^*}\mu_N$ and $\mu_N = \frac{e}{2m_p}$ is the nuclear magneton.

We can calculate μ* using $1/a_0$=120*Mev*

$$\mu^* = \frac{3}{a_0} = 3\cdot 0.12 = 0.36 Gev?$$

Experimentally μ* =0.344*Gev* which is found by fitting the nucleon magnetic moments with the net sum of its quarks magnetic moments[3].

We found the quark magnetic moments classically, now we try to find it relatively.

We start with massless Dirac equation with high energy quarks $P_\mu >> e_q A_\mu$ with A_μ is the electromagnetic field.

The Idea here is the shifting in the quark momentum $p\rightarrow\delta p+3/a_0$ (baryons)which associates with the static potential shifting $\sigma r\rightarrow\sigma a_0+\ldots$

The massless charged field Dirac equation:

$$i\gamma^\mu(\partial_\mu + ieA_\mu)\psi = 0$$

Writing $\psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$ so we can write:

$$\begin{pmatrix}I & \\ & -I\end{pmatrix}(\varepsilon-e\phi)\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}-\begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix}\cdot\left(\vec{p}-e\vec{A}\right)\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=0 \text{ so:}$$

$$(\varepsilon-e\phi)\psi_1-\vec{\sigma}\cdot\left(\vec{p}-e\vec{A}\right)\psi_2=0 \quad and \quad -(\varepsilon-e\phi)\psi_2+\vec{\sigma}\cdot\left(\vec{p}-e\vec{A}\right)\psi_1=0$$

Define $\pi=P-eA$ so:

$$\vec{\sigma}\cdot\vec{\pi}\psi_2=(\varepsilon-e\phi)\psi_1 \quad and \quad \vec{\sigma}\cdot\vec{\pi}\psi_1=(\varepsilon-e\phi)\psi_2$$

From the second we have:

$$\psi_2=\frac{\vec{\sigma}\cdot\vec{\pi}}{(\varepsilon-e\phi)}\psi_1$$

The first becomes:

$$\frac{(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{\pi})}{(\varepsilon-e\phi)}\psi_1=(\varepsilon-e\phi)\psi_1$$

So we have:

$$(\varepsilon-e\phi)=\frac{(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{\pi})}{(\varepsilon-e\phi)}$$

Using the relation:

$$(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{\pi})=\vec{\pi}\cdot\vec{\pi}+i\vec{\sigma}\cdot(\vec{\pi}\times\vec{\pi}) \quad and \quad \vec{\pi}\times\vec{\pi}=ie\vec{B}$$

So we have:

$$(\varepsilon-e\phi)^2=\vec{\pi}\cdot\vec{\pi}+i\vec{\sigma}\cdot(ie\vec{B})$$

So:

$$(\varepsilon-e\phi)^2=(\vec{p}-e\vec{A})^2-e\vec{\sigma}\cdot\vec{B}$$

for high energy quarks $P>>eA$ we have:

$$\begin{aligned}
&\varepsilon-e\phi=((\vec{p}-e\vec{A})^2-e\vec{\sigma}\cdot\vec{B})^{1/2}\\
&so\\
&\varepsilon-e\phi=\left|\vec{p}-e\vec{A}\right|(1-\frac{e\vec{\sigma}\cdot\vec{B}}{(\vec{p}-e\vec{A})^2})^{1/2}\\
&\to\varepsilon-e\phi=\left|\vec{p}-e\vec{A}\right|(1-\frac{e\vec{\sigma}\cdot\vec{B}}{2(\vec{p}-e\vec{A})^2}+....)\\
&\to\varepsilon=\left|\vec{p}-e\vec{A}\right|+e\phi-\frac{e\vec{\sigma}\cdot\vec{B}}{2\left|\vec{p}-e\vec{A}\right|}+....
\end{aligned}$$

Because of the energy renormalization $\sigma r\to\sigma a_0+\sigma\delta r$ we make $P\to 3/a_0+\delta P$ (three quarks) so

$$\frac{e\vec{\sigma}\cdot\vec{B}}{2\left|\vec{p}-e\vec{A}\right|}\to\frac{e\vec{\sigma}\cdot\vec{B}}{2(3/a_0+\delta)}=\frac{e\vec{\sigma}\cdot\vec{B}}{2(3/a_0)}+......?$$

Setting * =3/a$_0$ then: $\frac{e\vec{\sigma}\cdot\vec{B}}{2\mu^*}=2\frac{e}{2\mu^*}\frac{\vec{\sigma}}{2}\cdot\vec{B}=g\frac{e}{2\mu^*}\vec{S}\cdot\vec{B}$?

So the quark magnetic moments μ_q; e=e$_q$ becomes $\mu_q=\frac{e_q}{2\mu^*}=\frac{e_q}{e}\frac{m_p}{\mu^*}\frac{e}{2m_p}=\frac{e_q}{e}\frac{m_p}{\mu^*}\mu_N$

Which is the same relation we found classically. Using 1/a$_0$=120*Mev*

$$\mu^*=\frac{3}{a_0}=3\cdot 0.12=0.36Gev?$$

Now we try to find the quarks angular momentum in the Hadrons and the Regge trajectories:
We assume the quark rotates in the ratio *r*, so the angular momentum for one quark J$_q$ is[5]

$$J_q=pr=\frac{p}{r}r^2 \qquad 8.8$$

If we assume F_c is the centripetal, therefore $F_c=\frac{Pc}{r}$: c=1 8.9

so the angular momentum J$_q$ becomes $J_q=F_c r^2$

Now if we assume that the potential σr is between two quarks so F$_c$=σ/2 therefore

$$J_q=\frac{\sigma}{2}r^2$$

If we put r=a so $J_q=\frac{\sigma}{2}a^2$ 8.5

Using the relation $a^2=\frac{N}{\sigma}=\frac{N}{K^2}$ we have $J_q=\frac{\sigma}{2}\frac{N}{\sigma}=\frac{1}{2}N$ 8.10

$$J_q=\frac{\sigma}{2}\frac{N}{\sigma}=\frac{1}{2}N \qquad 8.10$$

if we renormalize *N* to be the number of the excited quarks inside the hadrons near the ground states, so

$$from\ \varepsilon^2=N\sigma\rightarrow N=\frac{\varepsilon^2}{\sigma} \quad \text{therefore} \quad J=\sum J_q=\frac{1}{2}\sum\delta N=\frac{\varepsilon^2}{2\sigma}+\text{constant}$$

We can consider it as Regge trajectories relation[5] $J=\alpha'\varepsilon^2+\alpha_0$ with the slope $\alpha'=\frac{1}{2\sigma}$.

9. Quarks Field dual behavior(free, condensed)), scalar π particles

To remove the divergences in the path integral and make the Lagrange parameters Z_1, Z_2, Z_m,… constants, we suggested the modified propagators like:

$$\overline{\Delta}^{ab}_{\mu\nu}(k^2)=\frac{g_{\mu\nu}\delta^{ab}}{k^2-i\varepsilon}\left(1-\frac{a^2k^2}{1+a^2k^2}\right) \quad \textit{for gluons} \qquad 9.1$$

$$\overline{S}_{ij}(\not{p})=\frac{-\not{p}\delta_{ij}}{p^2-i\varepsilon}\left(1-\frac{a^2p^2}{1+a^2p^2}\right) \quad \textit{for quarks} \qquad 9.2$$

We saw that we can ignore the terms a^2p^2 /(1+ a^2p^2) and a^2k^2 /(1+ a^2k^2) at high energy chapter 5 but when the energy drops to limited energy, those terms take place, we can give them a physical meaning for that we search for the corresponding terms in the Lagrange. To do that we find the role of those terms in the Feynman diagrams, in self energies, quarks gluons vertex,…

We find that the terms a^2p^2 /(1+ a^2p^2) and a^2k^2 /(1+ a^2k^2) can be related to pairing quark-antiquark appears as scalar particles with mass 1/a and charges -1, 0, +1 we can interrupt these particles as pions.

That appears in the particles–antiparticles composition in Feynman diagrams which mean for the fields, there is dual behavior(free, composition), that field dual behavior lets to possibility for separating the particles(free) and possibility for composition them, so the dual behavior of the fields is elementary behavior.

But for the composed particles they must associate with negative potential to survive long.

Like many fermions condensation with spin zero, which are described by Klein Gordon equation because it is impossible(in that case) to describe them by Dirac equation, so the dual behavior is elementary behavior.

As we saw before the smallest value for a is $a_0 \neq 0$ that is because of the negative potential u_0 which lets the quarks located in certain regions, so the perturbation would be broken. That occurs for any interaction when $E+u<0$, $E>0$ (like Higgs field) in that case some of the free particles would be condensed and fill the negative potential and the others stay free, so we have dual behavior (free particles and condensed particles). we will try to see that using above propagators.

The quark self-energy is:

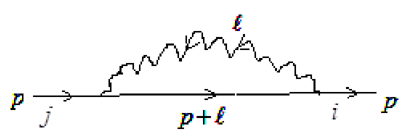


Diagram 9.1
represents the quark self-energy
for the strong interaction

$$i\Sigma_{ij}(\not{P}) = \int \frac{d^4\ell}{(2\pi)^4}[ig\gamma^\mu T^a_{ik}\frac{\overline{S}_{kl}(\not{P}+\not{\ell})}{i}ig\gamma^\nu T^b_{lj}]\frac{\overline{\Delta}^{ab}_{\mu\nu}(\ell^2)}{i}$$
$$= g^2T^a_{ik}T^b_{lj}\int\frac{d^4\ell}{(2\pi)^4}[\gamma^\mu\frac{(-\not{P}-\not{\ell})\delta_{kl}}{(p+\ell)^2}\gamma^\nu]\frac{g_{\mu\nu}\delta^{ab}}{\ell^2}$$

So we have:

$$i\Sigma_{ij}(\not{P}) = g^2T^a_{ik}T^a_{kj}\int\frac{d^4\ell}{(2\pi)^4}[\gamma^\mu\frac{(-\not{P}-\not{\ell})}{(p+\ell)^2}\gamma^\nu]\frac{g_{\mu\nu}}{\ell^2}$$

$$= g^2C(R)\delta_{ij}\int\frac{d^4\ell}{(2\pi)^4}[\gamma^\mu\frac{(-\not{P}-\not{\ell})}{(p+\ell)^2}\gamma_\mu]\frac{1}{\ell^2}$$

$$\textit{The sum:}\quad i\Sigma_{ij}(\not{P}) = 2g^2C(R)\delta_{ij}\int\frac{d^4\ell}{(2\pi)^4}\frac{(-\not{P}-\not{\ell})}{(p+\ell)^2}\frac{1}{\ell^2}$$
$$\textit{where}: \gamma^\mu(-\not{P}-\not{\ell})\gamma_\mu = 2(-\not{P}-\not{\ell})$$

Now we use the gluon modified propagator:

$$\overline{\Delta}^{ab}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}\delta^{ab}}{k^2-i\varepsilon}\left(1-\frac{a^2k^2}{1+a^2k^2}\right)$$

So we have addition term in quark self-energy :

$$i\Sigma_{ij}(\not{P}) = 2g^2 C(R)\delta_{ij}\int \frac{d^4\ell}{(2\pi)^4}\frac{(-\not{P}-\not{\ell})}{(p+\ell)^2}\frac{1}{\ell^2}\left(1-\frac{\mathrm{a}^2\ell^2}{1+\mathrm{a}^2\ell^2}\right)$$

So we separate it to two parts:

1-Quark−gluon part:

$$i\Sigma_{ij}(\not{P}) = 2g^2 C(R)\delta_{ij}\int \frac{d^4\ell}{(2\pi)^4}\frac{(-\not{P}-\not{\ell})}{(p+\ell)^2}\frac{1}{\ell^2}$$

2-pairing quarks part:

$$i\Sigma_{ij}(\not{P}) = 2g^2 C(R)\int \frac{d^4\ell}{(2\pi)^4}\frac{(-\not{P}-\not{\ell})\delta_{ij}}{(p+\ell)^2}\frac{1}{\ell^2}\left(-\frac{\mathrm{a}^2\ell^2}{1+\mathrm{a}^2\ell^2}\right) = 2g^2 C(R)\int \frac{d^4\ell}{(2\pi)^4}\frac{(\not{P}+\not{\ell})\delta_{ij}}{(-p-\ell)^2}\frac{1}{(-\ell)^2+1/\mathrm{a}^2} \qquad 9.3$$

It appears in the pairing part there is a scalar field propagator:

$$\frac{1}{i}\frac{1}{\ell^2+1/\mathrm{a}^2}$$

which is scalar particle propagator with mass 1/a , to preserve the charges, spin,… this particle must be condensed of quark−antiquark so we have addition diagram:

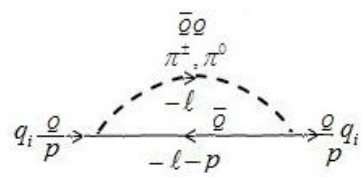


Diagram 9.2
Appears the scalar particle π in the quark self-energy diagram

$$\begin{aligned} i\Sigma_{ij}(\not{P}) &= 2g^2 C(R)\int \frac{d^4\ell}{(2\pi)^4}\frac{(\not{P}+\not{\ell})\delta_{ij}}{(-p-\ell)^2}\frac{1}{(-\ell)^2+1/\mathrm{a}^2} \\ &= 2(ig_s)^2 C(R)\int \frac{d^4\ell}{(2\pi)^4}\frac{\bar{S}_{ij}(-\not{P}-\not{\ell})}{i}\frac{-i}{(-\ell)^2+1/\mathrm{a}^2} \end{aligned} \qquad 9.4$$

So we can think that particle is the pion $\bar{q}q$ so we must add a new interaction term to the quarks Lagrange:

$$\Delta L = ig_\phi \varphi \bar{Q}Q \quad where \quad g_\phi = g_s\sqrt{2C(R)} \qquad 9.5$$

To satisfy the flavor symmetry, the scalar field $\varphi = \pi \equiv \bar{q}_i q_j$ becomes charged. For two flavors q_i , q_j we write the quarks field like $Q=(q_i \;\; q_j)^{\mathrm{T}}$ so

$$\Delta L = ig_{\pi q}\pi^a \bar{Q}T_2^a Q \quad where \quad \pi^a T_2^a \to \pi^0, \pi^-, \pi^+ \qquad 9.6$$

Which satisfies the quarks flavor invariance, so we can think that the field dual behavior associates with the flavor symmetry.

In general, when a→0 these pairing particles are removed, for non-strong interaction that is removed easily but for the interactions $E+u<0$, $E>0$ (like the massless charged particles) this pairing wouldn't be removed, that because of the energy renormalization where the negative energy is removed and have right states with positive energy and zero energy vacuum. In this case $E+u<0$ the value 1/a=m would be fixed and we have pairing particles(become condensed) associated with the free particles so we have dual behavior: particles and condensed particles in the case $E+u<0$.

For the quarks $a \to a_0 \neq 0$ smallest value of a with it we have right states, so $m=1/a_0$ is the pion mass we set $1/a_0 = 0.12Gev \approx$ pion mass.

Same thing we can find in quark−quark−gluon vertex:

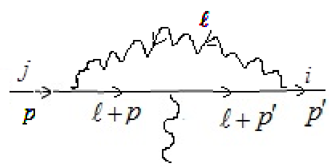


Diagram 9.3
quark-quark-gluon vertex

so we have addition diagram:

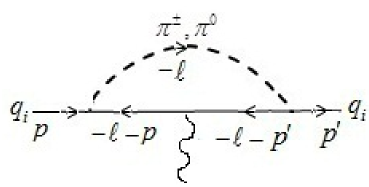
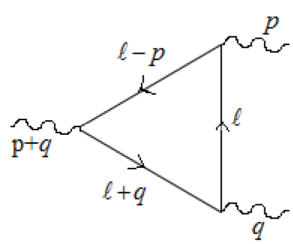


Diagram 9.4
Appears the scalar particle π in the quark-quark-gluon vertex for the strong interaction

There is some think different with three− photon vertex with single Weyl field $P_L\psi$:

Diagram 9.5
three photons vertex with single Weyl field



+.....

In that diagram when we use the propagators for electrons and photons instead of quarks and gluons then generate the result:

$$\bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2 - i\varepsilon}\left(1 - \frac{a^2k^2}{1+a^2k^2}\right) \qquad 9.7$$

$$\bar{S}(\not{p}) = \frac{-\not{p}}{p^2 - i\varepsilon}\left(1 - \frac{a^2p^2}{1+a^2p^2}\right) \qquad 9.8$$

With these propagators the axial current is conserved as we saw before so we expect the chiral symmetry is satisfied:

$$J^{\mu 5} = \bar{\psi}\gamma^{\mu}\gamma^{5}\psi \quad , \quad \partial_{\mu}J^{\mu 5} = 0 \qquad 9.9$$

The vertex of the three−photon and single Weyl field, from the diagram we have:

$$iV^{\mu\nu\rho}(p,\, q,\, r) \;=\; (-1)(ig)^3\left(\frac{1}{i}\right)^3 \int \frac{d^4\ell}{(2\pi)^4}\frac{N^{\mu\nu\rho}}{(\ell-p)^2\,\ell^2\,(\ell+q)^2} \;+\ldots\ldots$$

with

$$N^{\mu\nu\rho} = Tr[(-\not{\ell}+\not{p})\gamma^{\mu}\;(-\not{\ell})\gamma^{\nu}(-\not{\ell}-\not{q})\gamma^{\rho}P_L]$$

In the integral that becomes:

$$N^{\mu\nu\rho} = -\frac{1}{2} Tr[(-\not\ell + \not p)\gamma \ (-\not\ell)\gamma^{\nu}(-\not\ell - \not q)\gamma^{\rho}\gamma^{5}]$$

So the vertex becomes:

$$iV^{\mu\nu\rho}(p, q, r) = \frac{1}{2}(ig)^3\left(\frac{1}{i}\right)^3 \int \frac{d^4\ell}{(2\pi)^4} \frac{Tr[(-\not\ell + \not p)\gamma^{\mu}(-\not\ell)\gamma^{\nu}(-\not\ell - \not q)\gamma^{\rho}\gamma^{5}]}{(\ell - p)^2 \ell^2 (\ell + q)^2} + \ldots\ldots$$

using the propagators 9.1 and 9.2 we have $p_{\mu}V^{\mu\nu\rho} = q_{\nu}V^{\ \nu\rho} = r_{\rho}V^{?\nu\rho} = 0$ so we have $\partial_{\rho}J^{\rho 5} = 0 \ : \ J^{\rho 5} = \bar{\psi}\gamma^{\rho}\gamma^{5}\psi$ as we found before.

Rewritten the vertex like:

$$iV^{\mu\nu\rho}(p, q, r) = \frac{1}{2}(ig)^3\left(\frac{1}{i}\right)^3 \int \frac{d^4\ell}{(2\pi)^4} \frac{Tr[(-\not\ell - \not q)\gamma^{\rho}\gamma^{5}(-\not\ell + \not p)\gamma^{\mu}(-\not\ell)\gamma^{\nu}]}{(\ell - p)^2 \ell^2 (\ell + q)^2} + \ldots\ldots$$

dropping the trace, we have:

$$iV^{\mu\nu\rho}(p, q, r) = \frac{1}{2} ig\left(\frac{1}{i}\right)^3 \int \frac{d^4\ell}{(2\pi)^4} \frac{(-\not\ell - \not q) ig\gamma^{\rho}\gamma^{5}(-\not\ell + \not p)\gamma^{\mu}(-\not\ell) ig\gamma^{\nu}}{(\ell + q)^2 (\ell - p)^2 \ell^2} + \ldots\ldots$$

Replacing the propagator $S(\not\ell + \not q) = \frac{(-\not\ell - \not q)}{(\ell + q)^2}$ with $\frac{(-\not\ell - \not q)}{(\ell + q)^2}\left(1 - \frac{a^2(\ell + q)^2}{a^2(\ell + q)^2 + 1}\right)$

So we have new term in the vertex $V^{\mu\nu\rho}$ and then new diagram:

$$iV^{\mu\nu\rho}(p, q, r) = \frac{1}{2}(ig)^3\left(\frac{1}{i}\right)^3 \int \frac{d^4\ell}{(2\pi)^4} \frac{(-\not\ell - \not q)\left(-a^2(\ell + q)^2\right)\gamma^{\rho}\gamma^{5}(-\not\ell + \not p)\gamma^{\mu}(-\not\ell)\gamma^{\nu}}{\left[a^2(\ell + q)^2 + 1\right](\ell - p)^2 \ell^2 (\ell + q)^2} + \cdots \qquad 9.10$$

it becomes:

$$iV^{\mu\nu\rho}(p, q, r) = \frac{1}{2}(ig)^3\left(\frac{1}{i}\right)^3 \int \frac{d^4\ell}{(2\pi)^4} \frac{(-\not\ell - \not q)(-a^2)\gamma^{\rho}\gamma^{5}(-\not\ell + \not p)\gamma^{\mu}(-\not\ell)\gamma^{\nu}}{\left[a^2(\ell + q)^2 + 1\right](\ell - p)^2 \ell^2} + \ldots.$$

$$= \frac{1}{2}(ig)^3\left(\frac{1}{i}\right)^3 \int \frac{d^4\ell}{(2\pi)^4} \frac{(\not\ell + \not q)\gamma^{\rho}\gamma^{5}(-\not\ell + \not p)\gamma^{\mu}(-\not\ell)\gamma^{\nu}}{\left[(\ell + q)^2 + \frac{1}{a^2}\right](\ell - p)^2 \ell^2} + \ldots\ldots$$

we can write that like:

$$iV^{\mu\nu\rho}(p, q, r) = \frac{1}{2}(ig)^3\left(\frac{1}{i}\right)^3 \int \frac{d^4\ell}{(2\pi)^4} \frac{-\gamma^{\rho}(\not\ell + \not q)\gamma^{5}(-\not\ell + \not p)\gamma^{\mu}(-\not\ell)\gamma^{\nu}}{\left[(\ell + q)^2 + \frac{1}{a^2}\right](\ell - p)^2 \ell^2} + \ldots \qquad 9.11$$

Where we used the property $(\not\ell + \not q)\gamma^{\rho} = -\gamma^{\rho}(\not\ell + \not q) - 2(\ell + q)^{\rho}$

So we have:

$$iV^{\mu\nu\rho}(p, q, r) = \frac{1}{2}(ig)\left(\frac{1}{i}\right)^3 \int \frac{d^4\ell}{(2\pi)^4} \frac{-ig\gamma^{\rho}(\not\ell + \not q)\gamma^{5}(-\not\ell + \not p)\gamma^{\mu}(-\not\ell) ig\gamma^{\nu}}{\left[(\ell + q)^2 + \frac{1}{a^2}\right](\ell - p)^2 \ell^2} + \ldots \qquad 9.12$$

Now we can consider the two vertexes $ig\gamma^{\rho}$ and $ig\gamma^{\nu}$ come from the product with fermion–photon–vertex, so we omit them and have:

$$iV^{\mu}(p,\,q,\,r)=\ \frac{1}{2}(ig)\left(\frac{1}{i}\right)^{3}\int\frac{d^{4}\ell}{(2\pi)^{4}}\frac{-(\not\ell+\not q)\gamma^{5}(-\not\ell+\not p)\gamma^{\mu}(-\not\ell)}{\left[(\ell+q)^{2}+\frac{1}{\mathrm{a}^{2}}\right](\ell-p)^{2}\,\ell^{2}}\ +\ldots\ldots \qquad 9.13$$

therefore we have:

$$iV^{\mu}(p,\,q,\,r)=\ \frac{1}{2}\int\frac{d^{4}\ell}{(2\pi)^{4}}\frac{\frac{1}{i}i(\not\ell+\not q)\gamma^{5}i\frac{1}{i}(-\not\ell+\not p)ig\gamma^{\mu}\frac{1}{i}(-\not\ell)}{\left[(\ell+q)^{2}+\frac{1}{\mathrm{a}^{2}}\right](\ell-p)^{2}\,\ell^{2}}\ +\ldots\ldots \qquad 9.14$$

That can be written like:

$$iV^{\mu}(p,\,q,\,r)=\ \frac{1}{2}\int\frac{d^{4}\ell}{(2\pi)^{4}}i(\not\ell+\not q)\gamma^{5}\frac{1}{i\left((\ell+q)^{2}+\frac{1}{\mathrm{a}^{2}}\right)}i\frac{(-\not\ell+\not p)}{i(\ell-p)^{2}}ig\gamma\ \frac{(-\not\ell)}{i\ell^{2}}\ \ +\ldots\ldots$$

That can be represented in the diagram:

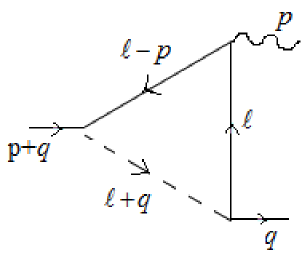


Diagram 9.6
Appears the scalar particle π in the three photons vertex with single Weyl field

So we must add new interaction terms to the Lagrange:

$$i\varphi\bar{\psi}\psi\ \ \ and\ \ \ (\partial_{\mu}\varphi)\bar{\psi}\gamma^{\mu}\gamma^{5}\psi \qquad 9.15$$

Where the field φ is scalar field with mass 1/a and propagator:

$$\Delta(p^{2})=\frac{1}{i}\frac{1}{p^{2}+1/\mathrm{a}^{2}} \qquad 9.16$$

That turned to be for the quarks where the length a takes the constant value a_0 that particle is the Pion with the mass $1/\mathrm{a}_0 \approx m_{pion}$, we found before $1/\mathrm{a}_0=0.12\,Gev$.

Because the strong interaction never distinguish between the flavors so the new interaction terms become $ig_{\pi q}\pi^{\alpha}\bar{Q}T_{2}^{\alpha}Q\ \ and\ \ g_{A}(\partial_{\mu}\pi^{\alpha})\bar{Q}T_{2}^{\alpha}\gamma^{\mu}\gamma^{5}Q$ then we have:

$$\Delta L_{\pi q}=ig_{\pi q}\pi\bar{q}_{i}q_{j}\ +g_{A}(\partial_{\mu}\pi)\bar{q}_{i}\gamma^{\mu}\gamma^{5}q_{j}$$
$$where\ \pi=\pi^{0},\pi^{-},\pi^{+}\ pions,\ with\ suitable\ flavors\ q_{i},q_{j} \qquad 9.17$$

q_i quark with flavor *i*.

Using the propagators 9.1 and 9.2 we had $p_{\mu}V^{\mu\nu\rho}=q_{\nu}V^{\mu\nu\rho}=r_{\rho}V^{\mu\nu\rho}=0$ (chapter 4), so we can have after some treatments $\partial_{\rho}J^{\rho 5}(x)=0$.

That current can be written like $J^{\rho 5}=J^{\rho 5}_{free}+J^{\rho 5}_{pairing}$ so $\partial_{\rho}J^{\rho 5}=\partial_{\rho}J^{\rho 5}_{free}+\partial_{\rho}J^{\rho 5}_{pairing}=0$ 9.18

For flavor symmetry we generate the current to $J^{\mu 5\alpha}_{free}=\bar{Q}\gamma^{\mu}\gamma^{5}T_{2}^{\alpha}Q$.

Therefore for gauge invariance, the chiral symmetry isn't satisfied $\partial_\rho J^{\rho 5\alpha}_{free} \neq 0$ but $\partial_\rho J^{\rho 5\alpha}_{free} + \partial_\rho J^{\rho 5\alpha}_{pairing} = 0$ where $J^{\rho 5\alpha}_{pairing}$ associates with the pairing particles-antiparticles behavior $\bar{q}_i q_j$ *, so that behavior associates with the flavor symmetry*. Therefore the gauge invariance and flavor invariance together satisfy the chiral symmetry.

The particles π^0, π^-, π^+ relate to the pairing behavior(particle-antiparticle), their interaction terms

$i\pi\bar{q}_i q_j$ *and* $(\partial_\mu \pi)\bar{q}_i \gamma^\mu \gamma^5 q_j$ with them the flavor chiral symmetry is satisfied(flavor invariance).

But we expect, the fields dual behavior takes place in negative potential. If there isn't negative potential the paired particles would not survive(never condense).

For the quarks, the case 0<*r*<a associates with negative potential *u* and *E*+*u*<0. Because the behavior of the strong interaction coupling constant at low energy (α_s high) we expect negative potential at low energy *E*+*u*<0 (*E*>*0*, *u*<0), so the quarks condense.

Because of the dual behavior of the quarks field which means for any two quarks interaction, the quarks composite and give scalar charged particles like the Pions π^-, π^0, π^+ and because of their quantized charges −1, 0, +1 we expect the hadrons charges also quantized –Q, –Q+1 ,…, 0, +1,…, +Q that quantization relates to the dual behavior of the quarks field in different hadrons, pairing quarks of different hadrons, so these condensed quarks; Pions, Kaons,… are shared between the hadrons, so put them together with the hadrons in groups, like the Pions −1, 0, +1 which can be inserted in SU(2) generators which can represent the proton−neutron pairing.

So the protons and neutrons Lagrange contains the terms $ig_{\pi N}\pi^\alpha \bar{N} T_2^\alpha N$ and $g_A(\partial_\mu \pi^\alpha)\bar{N} T_2^\alpha \gamma^\mu \gamma^5 N$ with the nucleon field $N = \begin{pmatrix} p \\ n \end{pmatrix}$.

10. The Quarks Plasma

We tried before to explain how the quarks are confinement at low limited energy we assumed some Ideas and the result was the condition *r*<a with that condition we have free quarks at high energies for the strong interaction where the length a is removed from the propagators, but it appears to be fixed at low limited energy, in the last section we showed that there is dual behavior for the quarks field, but when the length a is fixed, the result is scalar particles (pions) with mass $1/a_0$ at low limited energy and the result is the chiral symmetry separately breaking(last section).

We tried to give the length a physical meaning (quarks field dual behavior) also it appears in the quark-quark strong interaction (gluons exchanging) potential $U(r)_{r<a}$ so it indicates to interaction strength. That is because, the behavior of the length a is like the behavior of the coupling constant α_s.

That potential appears at low energy and absorbs the quarks energy and freezes them in the Hadrons, fermions hadrons and scalar hadrons.

We try here to use the statistical Thermodynamics to show how the free quarks disappear at low energies(low Temperatures) where the length a becomes fixed, so the chiral symmetry breaking and the quarks condensation.

One of the results is that the condensation phase eq10.8 not necessary associates with chiral symmetry breaking, that is, the chiral symmetry breaking appears at the end of the cooling process when the expanding and cooling are ended and the length a becomes fixed, therefore the chiral symmetry breaking occurs and the pions become massive $m = 1/a_0$.

We start with the massless quarks, their energy in volume *V*:

$$E = c\int_{a^3} d^3r\int_0^\infty d\varepsilon g(\varepsilon)\varepsilon \frac{1}{e^{\beta(\varepsilon-\mu(r))}+1} \quad : g(\varepsilon) = g_q \frac{V}{2\pi^2}\varepsilon^2 \qquad 10.1$$

$$where\ \mu(r) = \mu_0 + u(r)\ \ with\ \ u(r) = -\frac{4\alpha_s}{3r}\left(1-e^{-r/a}\right)$$

Here we inserted the quark-quark strong interaction potential *U(r)* in the chemical potential (for decreasing the quarks energies, as we think, the quarks potential absorbs the quarks energies and make them condense, phase changing) and because *r*<a we integrate over the volume a^3: *r* is the distance between the interacted quarks. We can absorb $4\alpha_s/3 \to \alpha_s$.

The constant c is determined by the comparing with free quarks high energy where the potential $U(r) \to 0$ and $\alpha_s \to 0$(decoupling) at high energies, so the length $a \to 0$ that is as we said before, the behavior of the length a is like the behavior of the coupling constant g_s therefore the quarks become free at high energies.

By integrating over the energy(*Maple program*) we have:

$$E = cg_q\frac{V}{2\pi^2}\int_{a^3} d^3r\int_0^\infty d\varepsilon\frac{\varepsilon^3}{e^{\beta(\varepsilon-\mu(r))}+1} = cg_q\frac{V}{2\pi^2\beta^4}\int_{a^3} d^3r\left[\frac{7\pi^4}{60}+\frac{\pi^2}{4}u_0(r)^2+\frac{1}{4}u_0(r)^4+6\sum_{k=1}^{\infty}\frac{(-1)^k e^{-k\beta\mu(r)}}{k^4}\right]$$

Where $u_0(r)=\beta\mu(r)=\beta(\mu_0+u(r))$.

by integrating over *r* (the distance between the interacted quarks) we have:

$$E = cg_q\frac{2Va^3}{\pi x^4}\left[3.78+(\beta\mu_0)^2\left(0.82-1.16\frac{\alpha_s}{a\mu_0}+0.41\left(\frac{\alpha_s}{a\mu_0}\right)^2\right)+(\beta\mu_0)^4\left(0.08-0.23\frac{\alpha_s}{a\mu_0}+0.25\left(\frac{\alpha_s}{a\mu_0}\right)^2-0.12\left(\frac{\alpha_s}{a\mu_0}\right)^3\right.\right.$$
$$\left.\left.+.02\left(\frac{\alpha_s}{a\mu_0}\right)^4\right)+6\sum_{k=1}^{\infty}\int_0^1 x^2dx\frac{(-1)^k e^{-k\beta\mu(x)}}{k^4}\right]$$

g_q is the quarks degeneracy number and $x=\beta\mu_0$.

Rewriting $\alpha_s/a= 2a\alpha_s/2a^2= \sigma a/2$. For more easy we write $\alpha_s/a\mu_0 = \sigma a/2\mu_0 = y$ in the energy relation. So rewriting the energy *E* as

$$E = cg_q\frac{2Va^3}{\pi x^4}\left[3.78+(\beta\mu_0)^2\left(0.82-1.16y+0.41y^2\right)+(\beta\mu_0)^4\left(0.08-0.23y+0.25y^2-0.12y^3+0.02y^4\right)\right.$$
$$\left.+6\sum_{k=1}^{\infty}\int_0^1 x^2dx\frac{(-1)^k e^{-k\beta\mu(x)}}{k^4}\right] \qquad 10.2$$

$$x=\beta\mu_0=\frac{\mu_0}{T}\to 0\ at\ high\ energy$$

To find the constant c we compare with quarks high energy where they are free massless particles:

$$E_{high} = g_q V\frac{7\pi^2}{240}T^4$$

When *T* is high, $x=(\mu_0/T)\to 0$ and $y\to 0$ therefore $\beta\mu(x)\to 0$ so we expand $e^{-k\beta\mu(x)}$ near $\beta\mu(x)=0$, we have:

$$E_{high} = cg_q\frac{2a^3V}{\pi x^4}\left[3.78-1.88+O(x,y)\right]\to cg_q\frac{2a^3V}{\pi x^4}1.9$$

$$\longrightarrow g_q\frac{7\pi^2V}{240}T^4 = cg_q\frac{2a^3V}{\pi x^4}1.9 \to c = \frac{\pi}{2a^3 1.9}\frac{7\pi^2}{240}\mu_0^4 \qquad 10.3$$

The energy becomes:

$$E=\frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^4 g_q\frac{V}{(\beta\mu_0)^4}\Big[3.78+(\beta\mu_0)^2\left(0.82-1.16y+0.41y^2\right)+(\beta\mu_0)^4\left(0.08-0.23y+0.25y^2-0.12y^3+0.02y^4\right)$$
$$+6\sum_{k=1}^{\infty}\int_0^1 x^2dx\frac{(-1)^k e^{-ku_0(x)}}{k^4}\Big]$$

Now we see the effects of the length a on the energy, at high energy, by fixing $x=\ \mu_0/T$ and varying y=σa/2 μ_0 <1:

$$E_{high}=\frac{1}{1.9}\frac{7\pi^2}{240}g_q V\mu_0^4x^{-4}\big[1.9+x(1.8-1.27y)+x^3(0.82-1.7y+1.24y^2-0.29y^3)+x^4(0.04-0.12y+0.13y^2$$
$$-0.07y^3+0.01y^4)\big]_{x=\beta\mu_0\to 0} \qquad 10.4$$

we expanded $e^{-k\beta\mu(x)}$ near $\beta\,\mu(x)=0$ so we have the following diagram:

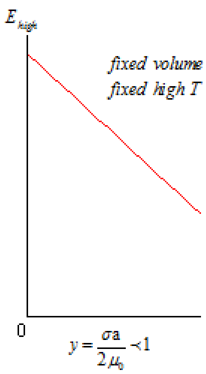


Diagram 10.1
representation the behavior of high energy quarks plasma with respect to the scale σa at high temperature in constant volume

We fixed the tension σ as we assumed before.

It appears from the diagram that the high energy quarks lose an energy when the length a increases although the temperature is fixed. That means, when the length a increases the number of the excited quarks decreases.

That is because of the attractive linear potential $\sigma r+\ldots$ between the quarks, that potential absorbs an energy, so the quarks are cooled faster by the expanding.

The fast cooling comes from the increasing the length a as we said before, the behavior of length a is like the behavior of the coupling constant α_s so when the energy dropped to lowest energy the length a increased extremely and that is fast cooling or extremely cooling, this is, when the particles try to spread away, so the length a increases and the result is induced cooling.

where the length a is the distances between the interacted quarks.

Or, when the quarks expand (increasing the distance a) they fast lose energy (extremely decreasing the Temperature T).

To determine the end, we search for a balance situation, such zero pressure, confinement condition,...

First we find the high energy pressure including the effects of the potential σa .

Starting from the general pressure relation:

$$p=-\frac{\partial}{\partial V}F \quad where\ F=-T\ln Z=-\frac{1}{\beta}\ln Z$$

We use here the relation:

$$\ln Z = c\int_{a^3} d^3r\int_0^\infty d\varepsilon g(\varepsilon)\ln\left(e^{-\beta(\varepsilon-\mu(r))}+1\right) \ : \ g(\varepsilon)=g_q\frac{V}{2\pi^2}\varepsilon^2$$

So the pressure becomes

$$P=\frac{1}{3}\frac{\partial}{\partial V}E$$

so for high energy $x=\beta\mu_0\to 0$ we have the pressure:

$$P_{high}=\frac{1}{3}\frac{\partial}{\partial V}E_{high}=\frac{\partial}{\partial V}\frac{1}{3\cdot 1.9}\frac{7\pi^2}{240}g_qV\mu_0^4x^{-4}\left[1.9+x(1.8-1.27y)+x^3(0.82-1.7y+1.24y^2-0.29y^3)\right.$$
$$\left.+x^4(0.04-0.12y+0.13y^2-0.07y^3+0.01y^4)\right]$$

Now is the key point, we want to include the potential effect on the pressure so we replace the volume V with the volume a^3 $\sim y^3$ so

$$P_{high}\to\frac{\partial}{\partial y^3}y^3\frac{1}{3\cdot 1.9}\frac{7\pi^2}{240}g_q\mu_0^4x^{-4}\left[1.9+x(1.8-1.27y)\ +x^3(0.82-1.7y+1.24y^2-0.29y^3)\right.$$
$$\left.+x^4(0.04-0.12y+0.13y^2-0.07y^3+0.01y^4)\right] \qquad 10.5$$

Which is represented in the diagram(without conditions on y or on the length a)

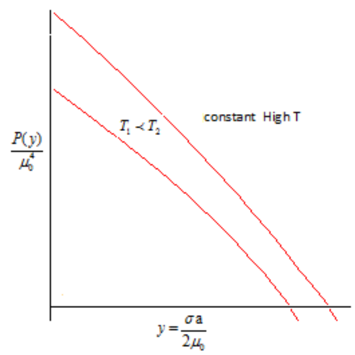


Diagram 10.2
representation the behavior of high energy quarks plasma pressure with respect to the scale σa at high temperature in constant volume

It is clear(without conditions on y) the pressure decreases, with increasing the length a (decreasing the quarks energy $-p^2$) until it becomes zero, then negative.

That becomes clear at low energy where there are conditions on y and so on the length a.

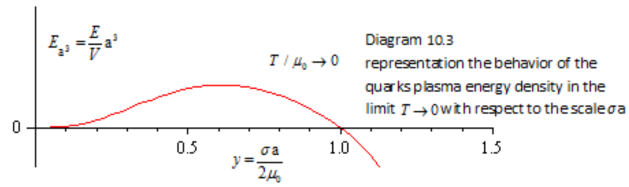
Now the low energy quarks, T→0 so $\beta\mu(x)\to\infty$ so $e^{-k\beta\mu(x)}\to 0$. The energy becomes:

$$E_{low}=\frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^4g_q\frac{V}{(\beta\mu_0)^4}\left[3.78+(\beta\mu_0)^2\left(0.82-1.16y+0.41y^2\right)+(\beta\mu_0)^4\left(0.08-0.23y+0.25y^2-0.12y^3+.02y^4\right)\right] \qquad 10.6$$

Making $x=T/\mu_0$

$$E_{low}=\frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^4g_qVx^4\left[\ \left[3.78+x^{-2}\left(0.82-1.16y+0.41y^2\right)+x^{-4}\left(0.08-0.23y+0.25y^2-0.12y^3+0.02y^4\right)\right]\right.$$

Now the key point, we want to show the effect of the potential σa on the energy so we see the behavior of the energy in the volume a^3 with respect to $y=\sigma a/\mu_0$ the diagram is:



That is extremely behavior after y=0.6 where the energy $(E/V)a^3$ decreases when the volume a^3 increases, the end in y=1 where the free quarks disappear when y>1

Now we can distinguish between the confinement and the chiral symmetry breaking, when y>0.6 there is confinement: extremely cooling, negative pressure.

but when reach y=1 there is chiral symmetry breaking where the length a becomes fixed, and from the quarks field dual behavior there are scalar charged particles with mass 1/a appear when the length a is fixed to have certain non-zero value a_0 .

Here the evidence for fixing the length a is the lowest limited quarks energy, that is as we said before, the behavior of the length a is like the behavior of the coupling constant α_s so when the quarks energy dropped (extremely cooling) the length a increases extremely to reach the highest value when y=1 which equivalents to smallest energy $E=0$ (the cooling end).

Another evidence for fixing the length a (chiral symmetry breaking) is the low energy pressure:

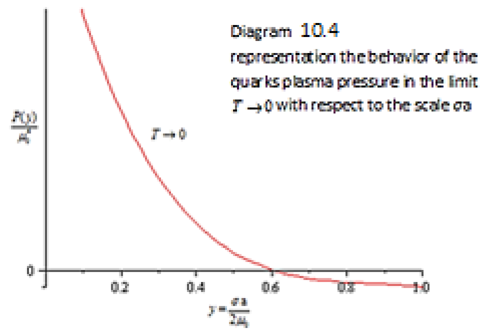
$$P_{low}=\frac{1}{3}\frac{\partial}{\partial V}E_{low}\to\frac{1}{3}\frac{\partial}{\partial y^3}\frac{E_{low}}{V}y^3$$

To include the potential effect we study the pressure using the volume $a^3 \sim y^3$ therefore

$$P_{low}\to\frac{1}{3}\frac{\partial}{\partial y^3}\frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^4 g_q y^3 x^4\left[3.78+x^{-2}\left(0.82-1.16y+0.41y^2\right)+x^{-4}\left(0.08-0.23y+0.25y^2-0.12y^3+0.02y^4\right)\right]: 10.7$$

$$\frac{P_{low}}{\mu_0^4}\to\frac{1}{9\cdot 1.9}\frac{7\pi^2}{240}g_q\Big[3\cdot 3.78x^4+3x^2\left(0.82-1.16y+0.41y^2\right)+0.08-0.23y+0.25y^2-0.12y^3+0.02y^4$$
$$+yx^2\left(-1.16+0.82y\right)+y\left(-0.23+0.5y-0.36y^2+0.08y^3\right)\Big]$$

so the low energy pressure becomes like the following diagram:



it is clear from that diagram when y>0.6 the quarks pressure becomes negative.

we expect the condensed quarks phase (confinement quarks) has positive pressure, so the preferred phase is the condensed quarks phase.

So when $y>0.6$ the quarks condense until y=1: $a \longrightarrow a_0 \approx 1/(120Mev)$* the quarks disappear, the scalar charged particles(Pions) appear instead of them, that is because of the quarks dual behavior(free-condensed quarks), but at low limited energy the condensed phase has a big chance instead the free Phase.

* the right values are 135–140 *Mev* the pions masses, but in our calculations(*Quarks Condensation phase, hadrons*) it is more suitable to use 120*Mev*(g_q=12), we can make 135–140 *Mev* but we have to change g_q(*Confinement phase*) so changing T_c to have same results(*Quarks Condensation phase, hadrons*).

10.1 Confinement phase

The confinement occurs(for any Temperature) when the attractive potential is higher than the quarks energy, so the quarks can't spread freely, they located in the space in certain distances between them in the .

They still have free particles behavior due to the fluctuations, but because of the fast cooling (extremely increasing the length a) they lose their energy and the Hadrons appear instead. Where the highest distance between the quarks is the length a_0 which is determined from the Hadrons.

for the protons and neutrons we found the energy σa_0=0.938*Gev* and we fixed $1/a_0$=0.12 $Gev \approx m_{pion}$ (pion mass). We expect the length $a=a_0$ is the same for all condensed quarks(Hadrons) so we think that it relates to the pion mass.

in general, we find the confinement phase condition:

$$\frac{E}{V}a^3 - \sigma a \prec 0$$

Dividing by μ_0 So

$$\frac{E_{low}}{2\mu_0 V}a^3 - \frac{\sigma a}{2\mu_0} \prec 0$$

And that condition becomes at low Temperature:

$$\frac{E_{low}}{2\mu_0 V}a^3 - y \prec 0 \qquad 10.8$$

So we have

$$\frac{1}{2}\frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^3 g_q x^4 \mathrm{a}^3\left[3.78 + x^{-2}\left(0.82 - 1.16y + 0.41y^2\right) + x^{-4}\left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4\right)\right] - y \prec 0$$

$$with\ x = \frac{T}{\mu_0} \to 0$$

it becomes

$$\frac{1}{2}\frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^3 g_q \mathrm{a}^3\left[3.78\cdot x^4 + x^2\left(0.82 - 1.16y + 0.41y^2\right) + \left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4\right)\right] - y \prec 0:\ \ x \to 0$$

so the critical $x_c y_c$ curve which separates the two phases:

$$\frac{1}{2}\frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^3 g_q \mathrm{a}^3\left[3.78\cdot x_c^4 + x_c^2\left(0.82 - 1.16y_c + 0.41y_c^2\right) + \left(0.08 - 0.23y_c + 0.25y_c^2 - 0.12y_c^3 + 0.02y_c^4\right)\right] - y_c = 0$$

Rewriting that like:

$$3.78x_c^4 + x_c^2\left(0.82 - 1.16y_c + 0.41y_c^2\right) + 0.08 - 0.23y_c + 0.25y_c^2 - 0.12y_c^3 + 0.02y_c^4 - \frac{240\cdot 1.9\cdot 2}{7\pi^2}\frac{y_c}{g_q(\mu_0\mathrm{a})^3} = 0$$

From the tension relation $\sigma = \frac{g_q\mu_0^2}{4\pi} \to \sigma\mathrm{a}^2 = \frac{g_q(\mu_0\mathrm{a})^2}{4\pi}$ we have

$$\mu_0\mathrm{a} = \frac{8\pi}{g_q}\frac{\sigma\mathrm{a}}{2\mu_0} = \frac{8\pi}{g_q}y \to \frac{8\pi}{g_q}:\ y \to 1$$

so

$$3.78x_c^4 + x_c^2\left(0.82 - 1.16y_c + 0.41y_c^2\right) + 0.08 - 0.23y_c + 0.25y_c^2 - 0.12y_c^3 + 0.02y_c^4 - \frac{0.00083g_q^2}{y_c^2} = 0$$

for quarks and anti-quarks the degeneracy $g_q = 2_{charge}\mathrm{x}2_{spin}\mathrm{x}3_{color} = 12$ so

$$2\cdot 3.78x_c^2 = -\left(0.82 - 1.16y_c + 0.41y_c^2\right) + \left[\left(0.82 - 1.16y_c + 0.41y_c^2\right)^2 - 4\cdot 3.78\left(0.08 - 0.23y_c + 0.25y_c^2 - 0.12y_c^3 + 0.02y_c^4 - 0.119y_c^{-2}\right)\right]^{1/2}$$

With the curve

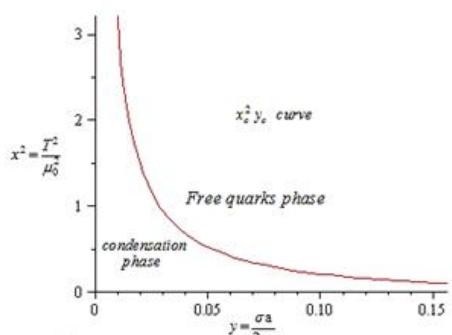


Diagram 10.5
shows the two quarks Phases under the extremely cooling $T \to 0$ when the expanding $0 \to \mathrm{a}$ occurs

For a point with $y<1$ in that curve there will be fast cooling to reach $y=1$: $a \longrightarrow a_0 \approx 1/(120Mev)$, so for $g_q=12$

$$\mu_0 a = \frac{8\pi}{g_q}\frac{\sigma a}{2\mu_0} = \frac{8\pi}{g_q} y \rightarrow \mu_0 a = \frac{8\pi}{12}$$

The length a_0 must be the same for all hadrons, it must be related to the pions mass $1/a_0 \approx$ pion mass, if we make $1/a_0=130mev$, we have $\mu_0 = \frac{1}{a}\frac{8\pi}{12} = 130\frac{8\pi}{12} = 272.27mev$

from x_c^2, y_c relation, for $y_c=1$ we find $x_c=0.41$ so from $x=T/\mu_0$ we have $T_c= \mu_0 x_c= 272.27\text{x}0.41 =111.4mev$ where the chiral symmetry breaking.

At that Temperature all the quarks are cooled and condensed in massive Hadrons.

10.2 Quarks Condensation phase, hadrons

We found that the quarks below the curve $x_c^2 y_c$ become confinement where the quarks energy becomes smaller than the interaction potential and the end is at $y_c=1$, $x_c<0.41$ where scalar charged particles with mass $1/a_0$ appear, we think those particles are the pions as we saw in the last section(quarks field dual behavior) they appear in the quarks diagrams, these diagrams prove that the pions consist of quark and antiquark.

From quarks field dual behavior, the quarks in different Hadrons can interact and form the pions and the result is the interaction between the hadrons by pions exchanging. And because the pions are charged -1, 0, +1 so the hadrons charges also must be quantized by -1, 0, 1 So the pions are inserted in SU(2) generators for hadrons pairing.

Here we assume that the confined quarks condense and give spin1/2 hadrons and pions, that occurs at $x_c<0.41$, $y_c=1$ point when the free quarks energy gets zero, so at that point all the quarks become condensed in hadrons with spin1/2 and 0 .

Because of the fast cooling, the quantum structure at low energy becomes same structure of the high energy, same spins, charges ratios, interactions,….

We assume, the condensation starts at high Temperature $x_c>>0.41$ with $y_c \rightarrow 0$ (condensation phase figure 10.5) the results are massless high energy Hadrons.

Then the cooling $y_c \rightarrow 1$, $x_c \rightarrow 0.41$ which is extremely cooling, at that point $y_c=1$ the pions become massive with $m=1/a_0$ (as we saw from the quarks dual behavior).

Due to the quarks field dual behavior, all hadrons (bosons or fermions) are interact by the pions exchanging (pairing and condensation quarks of different hadrons). Therefore when the pions become massive at $y_c=1$ all other Hadrons also become massive.

The condensation condition is $\frac{E}{V}a^3 - \sigma a \prec 0$ so the critical energy density is $\frac{\sigma a}{a^3}$ below it the quarks condense and then extremely cooled to $y_c=1$. So we expect that energy density is transferred to the produced hadrons and photons like

$$\frac{\sigma a}{a^3} \rightarrow \frac{E_{hadrons} + E_{photons}}{V} \quad \textit{below } x_c y_c \textit{ curve}$$

Or writing the densities $\varepsilon_f = \frac{E_f}{V}, \varepsilon_b = \frac{E_b}{V}$ *and* $\varepsilon_{ph} = \frac{E_{ph}}{V}$ for spin 1/2 hadrons(fermions), spin 0 hadrons (bosons) and photons densities. So

$$\frac{\sigma \mathrm{a}}{\mathrm{a}^3} \to \varepsilon_f + \varepsilon_b + \varepsilon_{ph}$$

Now the key point, because the cooling is extremely cooling, like to take all the particles (quarks) from high Temperature and put them at low Temperature, so the same structure at high energy will be at low energies, like the charges distribution, particles densities ratios, energy distribution, spins…

The high Temperature fermions(condensed quarks) density (massless $y_c \approx 0$ and ignoring the chemical potential):

$$n_f = \frac{N_f}{V} = g_f \frac{3\zeta(3)}{4\pi^2} T^3$$

And the high Temperature bosons(condensed quarks) density(massless $y_c \approx 0$):

$$n_b = \frac{N_b}{V} = g_b \frac{\zeta(3)}{\pi^2} T^3$$

Therefore the ratio n_f/n_b at high energy is:

$$\frac{n_f}{n_b} = \frac{3}{4}\frac{g_f}{g_b}$$

We assume the density n_b is the density of the pions.

So because of the dual behavior of the quarks field, the pion will interact with the spin 1/2 hadrons, the spin and charges are conserved, so the ratio n_f/n_b remains the same when the hadrons(condensed quarks) are extremely cooled to y_c=1.

At x_c<0.41, y_c=1 the pions(scalar hadrons) become massive particles with $m=1/a_0 \approx 120$*mev* (we need that value for our calculations below, the right value is 135 *Mev*), as we found before, so their energy will appear in their masses so we can write

$$\varepsilon_b = n_b m_b = g_b \frac{\zeta(3)}{\pi^2} T_c^3 \frac{1}{\mathrm{a}}$$

Where we assumed the bosons hadrons here are the pions

We found the critical point x_c=0.41 with y_c=1 so the temperature T_c=111.4*mev* if we consider the bosons (here pions) with three charges -1, 0, 1 so g_b=3 therefore the density n_b becomes(near T_c =111.4*mev*)

$$n_b = g_b \frac{\zeta(3)}{\pi^2} T_c^3 = 3 \cdot \frac{1.2}{\pi^2} (111.4)^3 Mev^3 \quad \text{massless yc<1 ,and } T \to Tc = 111.4 mev$$

When y_c=1 and T_c=111.4*mev* the pions become massive $m=1/a_0$ so their energy density becomes

$$\varepsilon_b = n_b m_b = g_b \frac{\zeta(3)}{\pi^2} T_c^3 \frac{1}{\mathrm{a}} = 3 \cdot \frac{1.2}{\pi^2} (111.4)^3 \cdot 120 Mev^4 \;\; : T \to T_c = 111.4 Mev$$

at that point the fermions density becomes

$$from \;\; \frac{n_f}{n_b} = \frac{3}{4}\frac{g_f}{g_b} \to n_f = \frac{3}{4} \cdot \frac{6}{3} n_b = \frac{3}{4} \cdot \frac{6}{3} \cdot 3 \cdot \frac{1.2}{\pi^2} (111.4)^3 \approx 7.5638 \cdot 10^5 Mev^3$$

The photons energy density

$$\varepsilon_{ph} = \frac{E_{ph}}{V} = g_{ph} \frac{\pi^2}{30} T^4$$

At y_c=1, T_c=111.4*mev* it equals

$$\varepsilon_{ph} = 2 \cdot \frac{\pi^2}{30}(111.4)^4 Mev^4$$

Now, below y_c=1, T_c=111.4*mev* the critical quarks energy density ε_c is totally transferred to $\varepsilon_f + \varepsilon_b + \varepsilon_{ph}$ so

$$\varepsilon_c = \frac{\sigma a}{a^3} = \varepsilon_f + \varepsilon_b + \varepsilon_{ph}$$

Therefore $$\frac{\sigma a}{a^3} = \varepsilon_f + 3 \cdot \frac{1.2}{\pi^2}(111.4)^3 \cdot 120 + 2 \cdot \frac{\pi^2}{30}(111.4)^4$$

We can write $\frac{\sigma a}{a^3} = \frac{\sigma a^2}{a^4}$

We had the relation $\sigma a^2 = \frac{8\pi}{6}$ so $\frac{\sigma a^2}{a^4} = \frac{8\pi}{6}(120)^4 mev^4$

Therefore we have $$\frac{8\pi}{6}(120)^4 mev^4 = \varepsilon_f + 3 \cdot \frac{1.2}{\pi^2}(111.4)^3 \cdot 120 + 2 \cdot \frac{\pi^2}{30}(111.4)^4$$

So $$\varepsilon_f = \frac{8\pi}{6}(120)^4 - 3 \cdot \frac{1.2}{\pi^2}(111.4)^3 \cdot 120 - 2 \cdot \frac{\pi^2}{30}(111.4)^4 \approx 7.0675 \cdot 10^8 Mev^4$$

We can find the energy average for these fermions:

$$\bar{E}_f = \frac{\varepsilon_f}{n_f} \approx \frac{7.0675 \cdot 10^8 Mev^4}{7.5638 \cdot 10^5 Mev^3} \approx 934 Mev$$

Which is very closed to proton and neutron masses, so we can think that these fermions are the baryons p^-, p^+, n where that energy appeared in the masses because of the suddenly fast cooling, extremely quarks cooling, the total quarks momentum is zero, therefore the condensed quarks have small kinetic energy. Also because the quarks confinement occurs only when the closed quarks try to spread away, so the confinement quarks have opposite momentums. Therefore the produced Hadrons are with small kinetic energies. So the energy 934 *Mev* appears in the masses.

we try to calculate the ratio N_q/N_h using condensation phase relation like

$$N_q \delta\mu_q + N_h \delta\mu_h = 0$$

N_q the total quarks number(quarks and anti-quarks) which totally condense in the hadrons, N_h the total hadrons (fermions and bosons), μ_q the quarks chemical potential and μ_h the hadrons chemical potential.

We assumed before the relation for the quarks chemical potential

$$\mu(r) = \mu_0 + u(r) \quad with \quad u(r) = -\frac{\alpha_s}{r}\left(1 - e^{-r/a}\right)$$

$$so \quad \delta\mu_q(r) = u(r) = -\frac{\alpha_s}{r}\left(1 - e^{-r/a}\right)$$

The effect of that changing appeared in $y=\alpha_s/2a\mu_0 = \sigma a/2\mu_0$ in the results.

now for the hadrons we have

$$\delta\mu_h = -\frac{N_q}{N_h}\delta\mu_q = -\frac{N_q}{N_h}u(r)$$

So we have for the hadrons the same relations for the quarks, that if we consider the hadrons are massless, that is right for $x_c>0.41$ and $y_c<1$(in the condensation phase) so we have the chemical potential for the hadrons

$$\mu_h(r)=\mu_{0h}-u(r) \;\; with \;\; u(r)=-\frac{\alpha_s}{r}\left(1-e^{-r/a}\right)$$

therefore we replace $y \rightarrow -(N_q\mu_{0q} / N_h\mu_{0h})y$ in the quarks energy to get the hadrons energy. So the energy for the hadrons

$$E_{H,low}=\frac{1}{1.9}\frac{7\pi^2}{240}\mu_{0h}^4 g_h V x^4\left[3.78+x^{-2}\left(0.82+1.16\frac{N_q\mu_{0q}}{N_h\mu_{0h}}y+0.41\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^2 y^2\right)+x^{-4}\left(0.08+0.23\frac{N_q\mu_{0q}}{N_h\mu_{0h}}y\right.\right.$$
$$\left.\left.+0.25\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^2 y^2+0.12\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^3 y^3+0.02\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^4 y^4\right)\right] \qquad 10.9$$

When $x_c \rightarrow 0.41$, $y_c \rightarrow 1$ we have

$$E_{H,low}\sim\mu_{0h}^4\left[3.78\cdot\frac{(0.41)^4}{0.08}+\frac{(0.41)^2}{0.08}\left(0.82+1.16\frac{N_q\mu_{0q}}{N_h\mu_{0h}}+0.41\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^2\right)\right.$$
$$\left.+\left(1+\frac{0.23}{0.08}\frac{N_q\mu_{0q}}{N_h\mu_{0h}}+\frac{0.25}{0.08}\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^2+\frac{0.12}{0.08}\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^3+\frac{0.02}{0.08}\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^4\right)\right]$$

Assuming $\mu_{0h}=\mu_{0q}$ so

$$E_{H,low}\sim\mu_{0q}^4\left(4.02+5.31\frac{N_q}{N_h}+3.98\left(\frac{N_q}{N_h}\right)^2+1.5\left(\frac{N_q}{N_h}\right)^3+0.25\left(\frac{N_q}{N_h}\right)^4\right)$$

So we expect the hadrons chemical potential

$$\mu_h^4=\mu_{0q}^4\left(4.02+5.31\frac{N_q}{N_h}+3.98\left(\frac{N_q}{N_h}\right)^2+1.5\left(\frac{N_q}{N_h}\right)^3+0.25\left(\frac{N_q}{N_h}\right)^4\right)$$

We can calculate μ_h from the Fermi energy E_f from the average fermions energy, we had before

$$\bar{E}_f=\frac{\varepsilon_f}{n_f}\approx 934Mev$$

So for massless hadrons(fermions, $T>T_c$), the relation between the average fermions energy and the Fermi energy

$$\bar{E}_{fermion}=\frac{3}{4}E_{fermi}\rightarrow\mu_h=E_{fermi}=\frac{4}{3}\bar{E}_{fermion}=\frac{4}{3}\cdot 934Mev$$

So for μ_{0q}=272.27mev

$$\left(\frac{4}{3}\cdot 934\right)^4=(272.27)^4\left(4.02+5.31\frac{N_q}{N_h}+3.98\left(\frac{N_q}{N_h}\right)^2+1.5\left(\frac{N_q}{N_h}\right)^3+0.25\left(\frac{N_q}{N_h}\right)^4\right)$$

Not all quarks condense in fermions hadrons $N_{q\rightarrow f}$ part of them condense in the bosons hadrons $N_{q\rightarrow b}$ and part annihilate to photons $N_{q\rightarrow ph}$ therefore we write

$$N_q\rightarrow N_{q\rightarrow f}+N_{q\rightarrow b}+N_{q\rightarrow ph}$$

So $$\frac{N_q}{N_h} \to \frac{N_{q\to f} + N_{q\to b} + N_{q\to ph}}{N_h}$$

Because $\mu_{bosons} << \mu_{fermions}$ so from $N_q\delta\mu_q + N_h\delta\mu_h = 0$ 10.10

We get $$N_q\delta\mu_q + N_{hf}\delta\mu_{hf} + N_{hb}\delta\mu_{hb} + N_{hph}\delta\mu_{hph} = 0$$

It becomes $$N_q\delta\mu_q + N_{hf}\delta\mu_{hf} = 0 \quad or \quad N_q\delta\mu_q + N_f\delta\mu_f = 0$$

We take only N_f in the denominator $\frac{N_{q\to f} + N_{q\to b} + N_{q\to ph}}{N_h}$

Therefore it becomes $$\frac{N_{q\to f} + N_{q\to b} + N_{q\to ph}}{N_f}$$

We had the relation $$\frac{n_f}{n_b} = \frac{3}{4}\frac{g_f}{g_b}$$

So for bosons(pions here, two quarks) $\frac{N_f}{N_b} = \frac{3}{4}\cdot\frac{6}{3} = \frac{3}{2}$ and $N_{q\to b}$=2N_b and for the photons $\frac{N_f}{N_{ph}} = \frac{3}{4}\cdot\frac{6}{2} = \frac{9}{4}$ and $N_{q\to b}$=2N_b therefore we have

$$\frac{N_{q\to f} + N_{q\to b} + N_{q\to ph}}{N_f} = \frac{N_{q\to f}}{N_f} + \frac{N_{q\to b}}{N_f} + \frac{N_{q\to ph}}{N_f} = \frac{N_{q\to f}}{N_f} + \frac{2N_b}{N_f} + \frac{2N_{ph}}{N_f}$$

So $$\frac{N_{q\to f} + N_{q\to b} + N_{q\to ph}}{N_f} = \frac{N_{q\to f}}{N_f} + 2\cdot\frac{2}{3} + 2\cdot\frac{4}{9} = \frac{N_{q\to f}}{N_f} + \frac{20}{9}$$

Therefore we have $$\frac{N_q}{N_h} \to \frac{N_{q\to f}}{N_f} + \frac{20}{9}$$

In the equation $$\left(\frac{4}{3}\cdot 934\right)^4 = (272.27)^4\left(4.02 + 5.31\frac{N_q}{N_h} + 3.98\left(\frac{N_q}{N_h}\right)^2 + 1.5\left(\frac{N_q}{N_h}\right)^3 + 0.25\left(\frac{N_q}{N_h}\right)^4\right)$$

its solution is N_q/N_h=4.86 so

$$\frac{N_q}{N_h} = \frac{N_{q\to f}}{N_f} + \frac{20}{9} = 4.86 \to \frac{N_{q\to f}}{N_f} = 2.64$$

Because the fermions here must consist of odd number of quarks, the value 2.64 in $N_{q\to f}/N_f$ =2.64 is closed to be three quarks condensation(baryons $N_{q\to f}/N_f$=3).

10.3 The nuclear compression

we saw that the cooled hadrons have high density, so there is high pressure, that pressure makes influence δa so δy near y=1 or it makes y=1+δy, so the cooled quarks inside the hadrons fluctuate and give pions, that depends on the energy, if the energy is high then they give heavy hadrons, that processes lets the interacted hadrons lose an kinetic energy and form the pions. These pions rise the hadrons chemical potential .

Because the number of quarks increases although the hadrons are fixed, therefore the hadrons energy decreases and they can't spread away. We can see how the chemical potential of the interacted hadrons changes under the fluctuation $\delta y \backsim \delta a$(due to the quarks interaction) from the condensation relation

$N_q\delta\mu_q + N_h\delta\mu_h = 0$ we have $\delta\mu_h = -N_q\delta\mu_q/N_h$
with the quarks chemical potential

$$\mu_q^4 = \mu_{0q}^4\left(4.02 - 5.31y + 3.98y^2 - 1.5y^3 + 0.25y^4\right) \ : \ T \to T_c = 111.4Mev$$

for the fluctuation δy we have

$$\delta\mu_h = -\frac{N_q}{N_h}\frac{\partial\mu_q}{\partial y}\delta y$$

from quarks chemical potential we find $\frac{\partial\mu_q}{\partial y} \prec 0$ so $-\frac{\partial\mu_q}{\partial y} \succ 0$ therefore we have

$\delta\mu_h = \frac{N_q}{N_h}\left(-\frac{\partial\mu_q}{\partial y}\right)\delta y \prec 0 \ when \ \delta y \prec 0$ which is the quarks compressing, when the hadrons collide together that lets to δy<0 so the hadrons loss energy and pions are created.

And when they try to extend(spread away) δy>0 so $\delta\mu_h$>0 they gain an energy, but because of the losing energy for the pions creating, there will be a negative potential, so that potential holds the hadrons in the nucleus at low energies.

For the interacted hadrons pressure we have the phase changing relation $V_q\delta P_q + V_h\delta P_h = 0 :\ V$ volume, we have

$$\delta P_h = -\frac{V_q}{V_h}\delta P_q = -\frac{V_q}{V_h}\frac{\partial P_q}{\partial y}\delta y$$

because $\partial P_q/\partial y<0 \rightarrow -\partial P_q/\partial y>0$ therefore $\delta P_h = -\frac{V_q}{V_h}\frac{\partial P_q}{\partial y}\delta y < 0 \ when \ \delta y < 0$

when the hadrons collide together δy<0 so their pressure decreases although their density increases and to satisfy the Pauli principle that lets to increasing their pressure not decreasing, so to solve that problem there must be negative potential.

$$\delta y = \left(-\frac{V_q}{V_h}\frac{\partial P_q}{\partial y}\right)^{-1}\delta P_h \;\; at\ y=1$$

So the hadrons chemical potential becomes

$$\delta\mu_h = \frac{N_q}{N_h}\left(-\frac{\partial\mu_q}{\partial y}\right)\left(-\frac{V_q}{V_h}\frac{\partial P_q}{\partial y}\right)^{-1}\delta P_h \;\; : y=1$$

We have

$$\delta\mu_h = \frac{N_q V_h}{N_h V_q}\left(\frac{\partial\mu_q}{\partial y}\right)\left(\frac{\partial P_q}{\partial y}\right)^{-1}\delta P_h \;\; : y=1$$

And

$$\left.\frac{\partial P_q}{\partial y}\right|_{x=0.41,\ y=1} = -10^8 Mev^4 \quad , \quad \left.\frac{\partial\mu_q}{\partial y}\right|_{x=0.41,\ y=1} = -44.01 Mev$$

So we have

$$\delta\mu_h(Mev) = \frac{N_q V_h}{N_h V_q}(-44.01)\left(-10^{-8}\right)\delta P_h = 44.01\cdot 10^{-8}\frac{n_q}{n_h}\delta P_h$$

by that we can find constant nuclear potential. Like to write

$$\delta\mu_h(Mev) = -V_0$$

V_0 is the potential for each hadron, therefore $V_0 = -44.01\cdot 10^{-8}\frac{n_q}{n_h}\delta P_h$; V_0 with unit *Mev* and δP_h with Mev^4 .

So when the hadron(fermions, like protons or neutrons) collide or join, their density increases $\delta\mu_h>0$ so their pressure rises $\delta P_h>0$, therefore there is a negative potential V_0 . At low energies that potential prevents them from spreading away.

11. The Big Bang

We assume that the universe was created from the vacuum with zero energy E=0 in each point in the space and dropped in each point to constant negative energy $-2\sigma a = u_0 < 0$ (transference to more stable deeper vacuum) with the vacuum potential:

$$\mathrm{U}(r) = u_0 + \sigma r \;\; : \;\; r < \mathrm{a} \qquad 11.0$$

This potential is similar to the quarks potential, so the universe is confined in the space, for right vacuum U(r)=0 .

We assumed before, the total quarks low energy is $E = \frac{N}{\mathrm{a}} = \sigma\mathrm{a} \succ 0$ but with the renormalization, removing the quarks negative interaction potential by the shifting $\mathrm{a}\rightarrow \mathrm{a}_0+\delta\mathrm{a}$ where a_0 is fixed (eqs 6.2, 6.3) the quarks energy becomes contained in the masses($m=\sigma\mathrm{a}_0$ universal cooling and condensation), so the most universal energy is contained in the masses, that is evidence to believe that the universal positive energy is associates with universal negative energy.

Now the key point, as for the quarks, we write u_0 in 11.0 like:

$$u_0 = -\frac{\alpha}{\mathrm{a}} = -\mathrm{a}\frac{\alpha}{\mathrm{a}^2}$$

α is like α_s we fix the value α/a^2 and relate it to a string tension like the quarks

$$\sigma = \frac{2\alpha}{a^2} = \text{constant} \rightarrow u_0 = -\frac{\sigma a}{2} = -\sigma a_0 \rightarrow 0 : a \rightarrow 0$$

the limit a→0 must associate with r→0 where there were just points in the space and the time stop.

The starting is the zero point energy $u_0=0$: a=0 so the universe was created from the vacuum E=0 and a=0 then a increased to take fixed nonzero value a_0 which is the end of the massless particles phase as for the quarks plasma.

We have positive point energy E_{point}: $E_p+u_0=U(r)=0$ so $E_p=-u_0>0$ in each point in the space. So the explosion occurred in each point in the space with constant energy E_p but we have infinity energy density $\frac{E_p}{r_p^3} : r_p \approx 0$, and due to the large pressure, the expand occurred, so the energy density becomes finite.

Because of $r_p \approx 0$ we have $\sigma r_p << E_p$, where σr_p is the energy of the created fields, the tension σ is constant.

So there is surplus energy $E_p-\sigma r_p$: r_p started from $r_p=0$ and increased to reach r_p =a with $E_p-\sigma a=0$ is the end of that process.

We assume that the negative point energy is hidden, it is not associated with any process, but maybe it induces the hadrons condensation, that can be seen if there is losing pressure associates with hadrons nuclear condensation as we will see.

To calculate the time for the spontaneous explosion and expand from the $r_p=0$ to $r_p=a_0=1/120(mev^{-1})$ the end of the massless particles phase, we assume that the expand occurs with the light speed c

$$a_0 = \frac{1}{120} = 0.008Mev^{-1} = 0.008 \cdot 1.973 \cdot 10^{-13} m$$
$$= 0.016 \cdot 10^{-13} m$$

The time for the explosion and expanding to a_0:

$$T = \frac{a_0}{c} = \frac{0.016 \cdot 10^{-13}}{3 \cdot 10^8} = 5.3 \cdot 10^{-24} \sec$$

11.1 The universal explosion and expanding

Now we try to explain how the universe exploded and expanded, we start from our assumptions we made before and find the Hubble parameter and try to find the dark energy and matter.

we found that the quarks expand to the length $a_0 \approx (120Mev)^{-1}$ then the hadrons appear instead.

we assume that the universe is created in every point in two dimensions space XY then the explosion in Z direction. That is by the quarks, in each point in XY flat the quarks were created and then they expand in each point XY to the length a_0 then the explosion in Z direction, the result is the universe in the space XYZ.

there wasn't universal explosion in the XY flat, the universal explosion was only in Z direction, in the flat XY there was extend due to the quarks expanding from r=0 to r = $a_0 \approx (120Mev)^{-1}$ the flat XY was infinity before the quarks expanding and it is infinity after that expanding, what happened is increasing in the number of the XY points, then the explosion in Z direction.

We assume both expanding(XY and Z) occurred with the light speed c .

To find the lost matter, dark matter and dark energy, we use the relation we found before:

$$V_0 = -44.01 \cdot 10^{-8} \frac{n_q}{n_h} \delta P_h : \ V_0 \text{ with } Mev, \ \delta P \text{ with } Mev^4$$

So $$\delta P_h = -\frac{n_h}{n_q}\frac{V_0}{44}10^8 = -\frac{N_h}{N_q}\frac{V_q}{V_h}\frac{V_0}{44}10^8 Mev^4$$

That changing in the pressure δP(independent on time) is related to the hadrons condensation phase to form the nucleuses, where the global pressure $\delta P=\delta P_h$ extremely dropped due to the nuclear attractive potential (make it the nuclear binding energy) $V_0=-8$ Mev[3]. This pressure δP_h is remained contained in the nucleuses, but globally isn't visible.

So there is hidden global pressure δP_h and we have to include that problem in the Friedman equations solutions, we notice that the nuclear attractive potential lets to increasing in the cooled hadrons densities. Therefore the decreasing in the hadrons pressure associated with the increasing of their densities (inside the nucleuses).

The result is excess in the local energy density, that effects appear in the laws, that is, the matter density appears to be larger than the right energy density. So there is neither dark matter nor dark energy, it is just global and local densities.

We start from the defining the scale parameter R(t) for the universe expanding we write[6]

$$ds^2 = -dt^2 + R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \qquad 11.1$$

We make k=0 flat Universe.

The Friedman equations can be written like[6]

$$-3\frac{\ddot{R}(t)}{R(t)} = 4\pi G_N(\rho + 3p) - \Lambda \qquad (1)$$

$$\frac{\ddot{R}(t)}{R(t)} + 2\frac{\dot{R}^2(t)}{R^2(t)} + 2\frac{k}{R^2(t)} = 4\pi G_N(\rho - p) + \Lambda \qquad (2)$$

$$\dot{\rho} = -3(\rho + p)\frac{\dot{R}(t)}{R(t)} \qquad (3)$$

If we sum (1) and (2) we have

$$-\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)} + 2\frac{k}{R^2(t)} = 4\pi G_N(\rho + p) \qquad (2')$$

setting k=0 it becomes

$$-\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)} = 4\pi G_N(\rho + p) \qquad (2')$$

Now we try to find the Hubble parameter $H(t) = \frac{1}{R(t)}\frac{dR(t)}{dt} = \frac{\dot{R}(t)}{R(t)}$

There are two different times $t<a_0$ free quarks phase and $t>a_0$ hadrons phase which is the expanding in Z direction.

That means there are two different spacetime Geometric, $t<a_0$ and $t>a_0$.

We start with $t<a_0$:

the velocity $\frac{dR_1}{dt} = \dot{R}(t)r$ equals to the light speed c=ħ=1 here, so

$$1 = \dot{R}(t)r \qquad t < a = a_0$$

Therefore

$$\dot{R}(t)=\frac{1}{r} \qquad t<\mathrm{a}=\mathrm{a}_0$$

So we can write

$$R(t)=\frac{t}{r} \qquad t<\mathrm{a}=\mathrm{a}_0$$

So the Hubble parameter becomes

$$H(t)=\frac{\dot{R}(t)}{R(t)}=\frac{1/r}{t/r}=\frac{1}{t} \qquad t<\mathrm{a}=\mathrm{a}_0 \qquad 11.2$$

Now we want to find the Hubble parameter in the phase $t>\mathrm{a}_0$

actually when the quarks expand from $r=0$ to $r=\mathrm{a}_0$ there will be infinity points expanding, so infinity expanding distance in XY space, but the expanding cannot excess the light speed c=1 therefore an explosion occurs in Z direction, so the universal explosion.

So the time $t=\tau$: $0\to\mathrm{a}_0$ for the free quarks phase will associate with t: $0\to\infty$ for the universal expanding, so we make the transformation

$$t=\frac{-c_0}{\tau-\mathrm{a}_0}\ : \quad \tau<\mathrm{a}_0 \qquad 11.3$$

Where c_0 constant, we can relate that relation to a spacetime Geometry. That means if the quarks space $r<\mathrm{a}_0=(120Mev)^{-1}$ is flat, so the hadrons space isn't, it is curved space, where we live.

It is convenient to consider the quarks space($r<\mathrm{a}_0$ large energy density) is curved not our space(low energy density).

Now we can find the Hubble parameter for the universe t: $0\to\infty$

For τ: $0\to\mathrm{a}_0$ we had $\quad H(\tau)=\dfrac{1}{R}\dfrac{dR}{d\tau}=\dfrac{1}{\tau}$

So we can write $\quad H(\tau)=\dfrac{1}{R}\dfrac{dR}{d\tau}=\dfrac{1}{R}\dfrac{dt}{d\tau}\dfrac{dR}{dt}$

From $t=\dfrac{-c_0}{\tau-\mathrm{a}_0}$ we have $\dfrac{dt}{d\tau}=\dfrac{t^2}{c_0}$ so

$$\frac{t^2}{c_0}\frac{1}{R}\frac{dR}{dt}=\frac{1}{\tau}=\frac{t}{\mathrm{a}_0 t-c_0}$$

That becomes $\quad \dfrac{1}{R}\dfrac{dR}{dt}=\dfrac{c_0}{t(\mathrm{a}_0 t-c_0)}$

Therefore the Hubble parameter becomes

$$H(t)=\frac{1}{R}\frac{dR}{dt}=\frac{c_0}{t(\mathrm{a}_0 t-c_0)}=\frac{1}{t\left(\frac{\mathrm{a}_0}{c_0}t-1\right)}=\frac{1}{t\left(c_0' t-1\right)}$$

The new constant $c_0'=\mathrm{a}_0/c_0$

Now we use our assumptions for the pressure effects on the energy density

When the hadrons are formed the nuclear interaction begin, one of the results is the increasing in the energy density, where the hadrons are cooled and condensed.

From the relation $\delta P_h = -\frac{n_h}{n_q}\frac{V_0}{44}10^8 = -\frac{N_h}{N_q}\frac{V_q}{V_h}\frac{V_0}{44}10^8 Mev^4$

for $V_0<0$ We find $\delta P_h>0$(independent on the time), that means for the Fermions, the energy density increased (Pauli principle).

So that density increasing plays a role in the equations, with it the calculated energy density is larger than the right energy density.

In Friedmann equations we have $\rho + p$ energy density plus pressure. To include the pressure effects on the energy density we write

$$\rho + p = \rho + \delta\rho - \delta\rho + p$$

We assume the pressure $P=\delta P_h$(independent on the time) effects on the energy density like

$$-\delta\rho + p = 0 \text{ so } \rho + p = \rho + \delta\rho - \delta\rho + p = \rho + \delta\rho$$

Then we write $\rho(t) = \rho + \delta\rho$

With $$\dot\rho(t) = \dot\rho + \frac{d}{dt}\delta\rho = \dot\rho \;:\; \frac{d}{dt}\delta\rho = \frac{d}{dt}\delta P_h = 0 \qquad 11.4$$

We assume the $\rho = \rho_{matter}$ is the right energy density of the visible matter, and the $\rho(t)$ is the local energy density which includes the nuclear interaction effects(the pressure effects, because that pressure is independent on the time so we can consider its effects on the energy density, increasing that density).

We make that in the Friedmann equations(2') and (3'), k=0

$$-\frac{\ddot R(t)}{R(t)} + \frac{\dot R^2(t)}{R^2(t)} = 4\pi G_N(\rho + p) = 4\pi G_N \rho(t) \qquad (2')$$

$$\dot\rho = -3(\rho + p)\frac{\dot R(t)}{R(t)} = -3\rho(t)\frac{\dot R(t)}{R(t)} = \dot\rho(t) \qquad (3')$$

using (2') and (3') we find the energy density using the Hubble parameter

$$H(t) = \frac{1}{R}\frac{dR}{dt} = \frac{1}{t(c_0' t - 1)} \qquad t > a_0$$

From (3') we have $\frac{-1}{3}\frac{R(t)}{\dot R(t)}\dot\rho(t) = \rho(t)$ so (2') becomes

$$-\frac{\ddot R(t)}{R(t)} + \frac{\dot R^2(t)}{R^2(t)} = \frac{-4\pi G_N}{3}\frac{R(t)}{\dot R(t)}\dot\rho(t)$$

That equation becomes $$\frac{\dot R(t)}{R(t)}\left(-\frac{\ddot R(t)}{R(t)} + \frac{\dot R^2(t)}{R^2(t)}\right) = \frac{-4\pi G_N}{3}\dot\rho(t)$$

Or $$H(t)\left(-\frac{\ddot R(t)}{R(t)} + H^2(t)\right) = \frac{-4\pi G_N}{3}\dot\rho(t) \qquad (2')$$

Using $$\frac{d}{dt}\frac{\dot R(t)}{R(t)} = \frac{\ddot R(t)}{R(t)} - \frac{\dot R^2(t)}{R^2(t)} \text{ so } \frac{d}{dt}H(t) = \frac{\ddot R(t)}{R(t)} - H^2(t)$$

Using $H(t) = \frac{1}{t(c_0' t - 1)}$

We have $$\frac{\ddot{R}(t)}{R(t)} = \frac{d}{dt} t^{-1}\left(c_0' t - 1\right)^{-1} + \frac{1}{t^2\left(c_0' t - 1\right)^2} = \frac{-2}{t^2\left(c_0' t - 1\right)}$$

The equation becomes

$$\frac{1}{t\left(c_0' t - 1\right)}\left(\frac{2}{t^2\left(c_0' t - 1\right)} + \frac{1}{t^2\left(c_0' t - 1\right)^2}\right) = \frac{-4\pi G_N}{3}\dot{\rho}(t) \;\rightarrow\; \frac{2}{t^3\left(c_0' t - 1\right)^2} + \frac{1}{t^3\left(c_0' t - 1\right)^3} = \frac{-4\pi G_N}{3}\dot{\rho}(t)$$

The solution is $$-\frac{c_0'^2}{c_0' t - 1} + \frac{1}{2t^2} + \frac{c_0'}{t} + \frac{c'^2}{2\left(c_0' t - 1\right)^2} = \frac{4\pi G_N}{3}\left(\rho(t) - \rho_0\right)$$

For finite results we put ρ_0=0 so

$$-\frac{c_0'^2}{c_0' t - 1} + \frac{1}{2t^2} + \frac{c_0'}{t} + \frac{c'^2}{2\left(c_0' t - 1\right)^2} = \frac{4\pi G_N}{3}\rho(t) \qquad t > \mathrm{a}_0$$

Now we calculate the contributions of the vacuum energy to the total energy using the cosmological constant Λ like

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{3H^2} = \frac{3H^2 - 8\pi G_N \rho(t)}{3H^2} = 1 - 2\frac{4\pi G_N}{3H^2}\rho(t)$$

Where the critical energy density $\rho_c = \frac{3H^2}{8\pi G_N}$

Using the density $\rho(t)$: $$-\frac{c_0'^2}{c_0' t - 1} + \frac{1}{2t^2} + \frac{c_0'}{t} + \frac{c'^2}{2\left(c_0' t - 1\right)^2} = \frac{4\pi G_N}{3}\rho(t)$$

We find

$$\Omega_\Lambda = 1 - 2t^2\left(c_0' t - 1\right)^2\left(-\frac{c_0'^2}{c_0' t - 1} + \frac{1}{2t^2} + \frac{c_0'}{t} + \frac{c'^2}{2\left(c_0' t - 1\right)^2}\right) = 1 - 2t^2\left(c_0' t - 1\right)^2 \frac{1/2}{t^2\left(c_0' t - 1\right)^2} = 1 - 1 = 0 \qquad 11.5$$

So the vacuum energy density is canceled, and the total energy is the matter energy $\Omega_{matter} = 1$ so $\frac{\rho(t)}{\rho_c} = 1$

Here $\rho(t)$ is $$\rho(t) = \rho_{matter} + \delta\rho$$

With $$\frac{4\pi G_N}{3}\rho(t) = -\frac{c_0'^2}{c_0' t - 1} + \frac{1}{2t^2} + \frac{c_0'}{t} + \frac{c'^2}{2\left(c_0' t - 1\right)^2} \qquad t > \mathrm{a}_0$$

And the constant $\delta\rho$ is $$\delta\rho = \delta P_h = -\frac{N_h}{N_q}\frac{V_q}{V_h}\frac{V_0}{44}10^8 Mev^4$$

To calculate $\delta\rho$(independent on the time) we assume the potential V_0 equals the nuclear potential -8*Mev* and assume $N_q/N_h \approx g_q/g_h \approx 6$

Now we try to find V_q/V_h the quarks volume $V_q = Sd_q$ and the hadrons volume $V_h = Sd_h$ as the figure

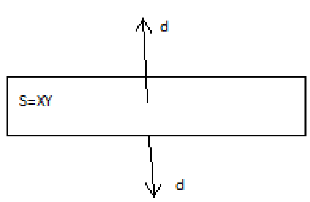


Diagram 11.1
represents the quarks creating and expanding in two dimension S=XY then the universal explosion in the third dimension Z in two opposite directions

Where the universal explosion in the $z=d$ direction.

If we assume the explosion speed is the same for both hadrons and quarks, light speed c=1, so for the quarks

$$H_q(t)=\frac{\dot{R}(t)}{R(t)}=\frac{1}{R(t)}=\frac{1}{d_q} \qquad 11.6a$$

For the hadrons $$H_h(t)=\frac{\dot{R}(t)}{R(t)}=\frac{1}{R(t)}=\frac{1}{d_h} \qquad 11.6b$$

So $$\frac{V_q}{V_h}=\frac{Sd_q}{Sd_h}=\frac{d_q}{d_h}=\frac{H_h}{H_q} \qquad 11.6c$$

We assume the H_h is the universal Hubble parameter which is

$$H=71km/s/mpc=2.3\cdot10^{-18}s^{-1}=2.3\cdot10^{-18}\cdot6.58\cdot10^{-22}Mev=151.34\cdot10^{-41}Mev$$

The quarks Hubble parameter $H_q=1/\tau=1/a_0=120Mev$

So we have $$\frac{H_h}{H_q}=\frac{151.34\cdot10^{-41}Mev}{120Mev}=1.26\cdot10^{-41}$$

Therefore $$\frac{V_q}{V_h}=\frac{H_h}{H_q}=1.26\cdot10^{-41}$$

The constant density changing $\delta\rho$: $$\delta\rho=\delta P_h=-\frac{N_h}{N_q}\frac{V_q}{V_h}\frac{V_0}{44}10^8Mev^4 \qquad 11.7$$

Becomes (with V_0 equals the nuclear potential -8*Mev*, nucleon binding energy)

$$\delta\rho=\delta P_h=-\frac{1}{6}\cdot1.26\cdot10^{-41}\cdot\frac{-8}{44}\cdot10^8Mev^4=381.8\cdot10^{-37}Mev^4$$

So the energy density $\rho(t)=\rho_{matter}+\delta\rho$ becomes

$$\rho(t)=\rho_{matter}+381.8\cdot10^{-37}Mev^4$$

we found $\Omega_\Lambda=0$ so $\rho(t)/\rho_c=1\longrightarrow\rho(t)=\rho_c$

experimentally, the critical density is

$$\rho_c=9.47\cdot10^{-27}kg/m^3=9.47\cdot10^{-27}\cdot4.29\cdot10^{-9}Mev^4\approx406\cdot10^{-37}Mev^4$$

therefore $$\rho(t)=\rho_c=406\cdot10^{-37}Mev^4=\rho_{matter}+381.8\cdot10^{-37}Mev^4$$

So we find the matter density $$\rho_{matter}=406\cdot10^{-37}Mev^4-381.8\cdot10^{-37}Mev^4=24.2\cdot10^{-37}Mev^4$$

The right baryonic matter energy density(BBM and CMB calculations) is

$$\rho_b=4.19\cdot10^{-31}g/cm^3\approx17.97\cdot10^{-37}Mev^4$$

There is no big difference between $\rho_{matter}=24.2*10^{-37}Mev^4$ which we found theoretically and $\rho_b=17.97*10^{-37}Mev^4$ which is the right.

The difference is $6.23*10^{-37}Mev^4$ may be related to the bosons matter like photons, mesons, …., but we can control this difference by changing the potential V_0 . like to replace $V_0\longrightarrow V_0+\delta V_0=-8-0.1305Mev$, with that, the difference is removed, we can relate the energy 0.13*Mev*(at least 0.13*Mev*) to the negative point energy(deeper vacuum), that if we assume that energy induces the hadrons condensation, or there is negative energy-positive energy potential −0.13*Mev*.

By that there is losing pressure eq11.7(equivalents to δV_0=−0.13*Mev* at least) associates with hadrons nuclear condensation(global cooling).

Therefore we can think that there is neither dark energy nor dark matter, it is just local and global matter densities. And all the matter is the visible matter.

Notice: the changing $\rho + p = \rho + \delta\rho - \delta\rho + p = \rho + \delta\rho$ is not to reform the Friedman equations(2')and(3'), it is just to know how the pressure effects on the energy density, instead of that, maybe we use the equivalent between the energy density and the pressure $\delta\rho=3\delta P$, but with that the ratios N_h/N_q and V_q/V_h would be changed to get the same result.

For more clear, we used the Friedman equations k=0 with the form

$$3\frac{\dot{R}^2(t)}{R^2(t)} = 8\pi G_N \rho + \Lambda \qquad (1')$$

$$-\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)} = 4\pi G_N(\rho + p) \qquad (2') \qquad 11.8$$

$$\dot{\rho} = -3(\rho + p)\frac{\dot{R}(t)}{R(t)} \qquad (3')$$

We wanted to include the effects of the increased energy density of the cooled hadrons(hidden pressure) on the solution of that equations, for that we can make

$$3\frac{\dot{R}^2(t)}{R^2(t)} = 8\pi G_N(\rho + \delta p) + \Lambda - 8\pi G_N \delta p$$

$$-\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)} = 4\pi G_N(\rho + \delta P + p - \delta P) \qquad 11.9$$

$$\frac{d}{dt}(\rho + \delta p) = -3(\rho + \delta p + p - \delta p)\frac{\dot{R}(t)}{R(t)}$$

Where $\delta p=\delta p_h>0$ is independent on the time.

So we have(for same Hubble parameter we had before)

$$\rho' = \rho + \delta p_h$$
$$p' = p - \delta p_h$$
$$\Lambda' = \Lambda - 8\pi G_N \delta p_h = 0$$

We can say ρ', p' and Λ' =0 are for the located matter, when the hadrons are cooled, they condense and locate in small volumes with high matter density, because of the strong nuclear attractive interaction, so their pressure extremely decreases. That pressure is contained(hidden) in the nucleus.

And ρ, p and Λ are the global measurements of the matter, the global measurements includes the large distances between the stars and planets. So we make them the right matter(visible matter).

Notice: Not all of these Ideas are contained in the References.

References

[1] Michael E. Peskin and Daniel V. Schroeder, An Introduction to Quantum Field Theory (Westview 1995).

[2] Mark Srednicki, Quantum Field Theory(University of California 2006).

[3] B. R. Martin, Nuclear and Particle Physics(University College London 2006).

[4] Lowell S. Brown, Quantum Field Theory (Cambridge 1994).

[5] Gerard 't Hooft, Introduction to string Theory, version 14-05-04(Institute for the Theoretical Physics, Utrecht University and Spinoza Institute).

[6] Matthias Blau, Lecture Notes on General Relativity(Universität Bern 2012).